Carnot Cycle

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Apps on Physics by Walter Fendt[2]

Abstract
The second law of thermodynamics puts on a limit on the engines we can construct. Ideal Carnot engine is the most efficient engine possible within the thermodynamical permit. However, real engines do not have a high efficiency due to various reasons. Here we have utilized the simulation with an engine having efficiency 20 % and with certain initial conditions. This simple simulation gives an idea about the processes constituting the cycle and flow of heat and work during each cycle. This simulation can be further used to study engines with different initial conditions.

1 Aim of the experiment

To strengthen the understanding of cyclic processes and engines utilizing the **Apps on Physics**. The following points will be touched upon:

1. Understanding the four stages of a Carnot cycle
2. Discuss the P-V and T-S diagrams
3. Calculate the efficiency of Carnot engine for a given setup

2 Introduction

2.1 Entropy and the Second Law of Thermodynamics

Conservation of energy does not define the direction of a process, yet some processes occur spontaneously, whereas others do not. For example, splattering of an egg is spontaneous but the splatter forming back the egg is not, but both have the same amount of energy associated with them. This propels us to define a physical property called *Entropy*. Entropy differs from energy in that entropy does not obey a conservation law. The energy of a closed system is conserved; it always remains constant. For irreversible processes, the entropy of a closed system always increases.[3] Mathematically, the change in entropy* is given by,

$$\Delta S = \frac{\Delta Q}{T} \tag{1}$$

Now, the second law of thermodynamics states that the change in a closed system entropy must be always greater or equal to zero. i.e. $\Delta S \geq 0$. Equality holds only for reversible process such as the cyclic process.[1]

*Actual value of entropy can not be measured. Only the change is measured by various experiments.

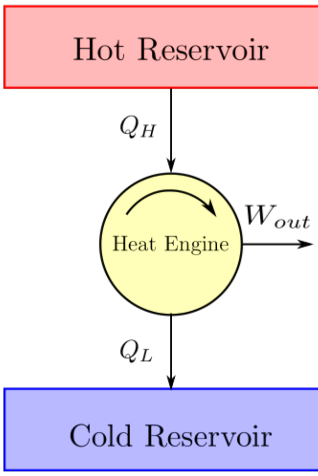


Figure 1: Schematic diagram of a heat engine. Source is at the top and sink is at the bottom. Credits-codecalculation

2.2 Carnot Cycle

The first recorded heat engine dates back to 50 AD designed by Hero of Alexander. The main concept of heat engines is to convert heat into work using various thermodynamic processes on a working substance, usually a gas. The Carnot cycle, an idealized engine, has a four-step cycle and an ideal gas as a working substance. All the heat engines have a similar structure with source, engine, and sink.
The Carnot cycle, designed by Nicolas Léonard Sadi Carnot in 1824, has four processes namely:

1. Isothermal expansion
2. Isentropic/Adiabatic expansion
3. Isothermal compression
4. Isentropic/Adiabatic compression

Each process is carried out in infinitesimally small steps keeping the whole process reversible. However, real engines are never ideally efficient due to mechanical friction and quasi-static property of the process.[1]

2.3 Efficiency of Heat Engines

In a heat engine, the working substance absorbs some heat Q_H from the hot reservoir (source), uses some fraction of it W_{out} in doing work, and the rest $Q_L$is given to the cold reservoir (sink). Using the conservation of energy,

$$Q_H = W_{out} + Q_L \tag{2}$$

This amount of work W_{out} is produced in one cycle of the engine and is converted to mechanical work. The efficiency of the engine (η) is given by,

$$\eta = \frac{Work\ done\ during\ a\ cycle}{Heat\ supplied\ to\ the\ gas} \tag{3}$$

$$\eta = \frac{W_{out}}{Q_H} = 1 - \frac{Q_L}{Q_H} \tag{4}$$

Now, specifically for a Carnot cycle, using the work-done in each process, we can derive the following relation,

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \tag{5}$$

where, Temperature T is in Kelvin.[4] Thus, putting eq (5) in eq (4) the efficiency of the Carnot cycle can be written as,

$$\eta = 1 - \frac{T_L}{T_H} \tag{6}$$

3 Methodology

3.1 Initializing the Parameters

The application provided by *Apps on Physics*[2] is utilized in this experiment to visualize the working of the engine. The interface of the app *Carnot Cycle* takes different inputs. We have taken certain values for easier calculations. The working substance used here is a diatomic ideal gas with adiabatic index (γ) = 1.40. 1 mole amount of gas is used for the cycle and shown as green inside the system in figure 2.
The simulation allows us to access the whole picture of the processes taking place in one interface. There are other two options to have a closer look at the P-V diagram and T-S diagram.

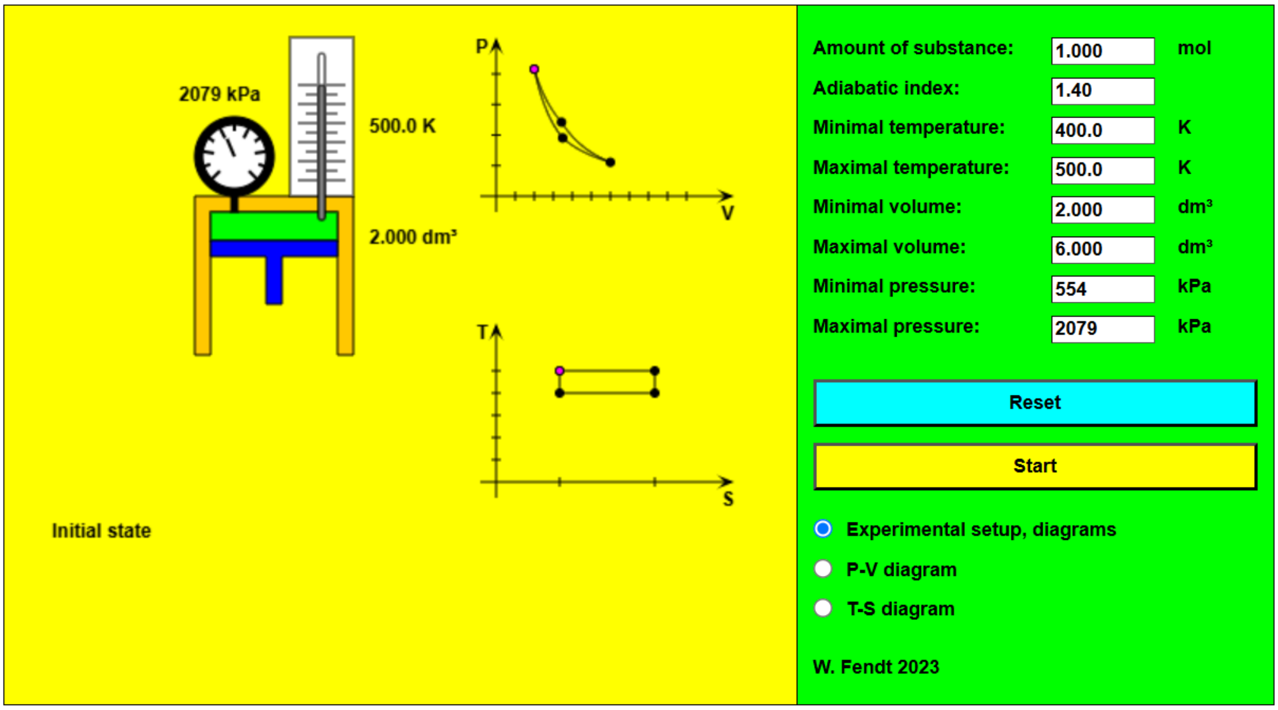


Figure 2: Initial values of various parameters shown on the left. The description of the Carnot engine is on the left.

3.2 Gathering Data

We have used all the available options to gather data from the simulation. The main aim of explaining the four processes of the Carnot cycle is attempted by abstracting a pictorial representation of each cycle from the app.
We have used specific values of the independent parameters and the app generates the values of pressure. For fixed values of volume, temperature, and amount of substance the app generates the values of pressure using the ideal gas equation:

$$PV = nRT \tag{7}$$

where, R is the Universal gas constant.
The plots available in the app are also used to give the reader a graphical view of the Carnot cycle. Different values of the initial conditions can be put in the app to see various paths and areas of the cycle produced.

4 Results

4.1 The Four-Stroke Engine

The four processes of the Carnot cycle can be seen in the figure 3. The working substance and the piston are colored green and blue respectively. The description of the processes are given as:

Isothermal expansion: The process is carried out in a closed system with constant temperature i.e. 500 K as the source. The gas is allowed to expand with a decrease in pressure.

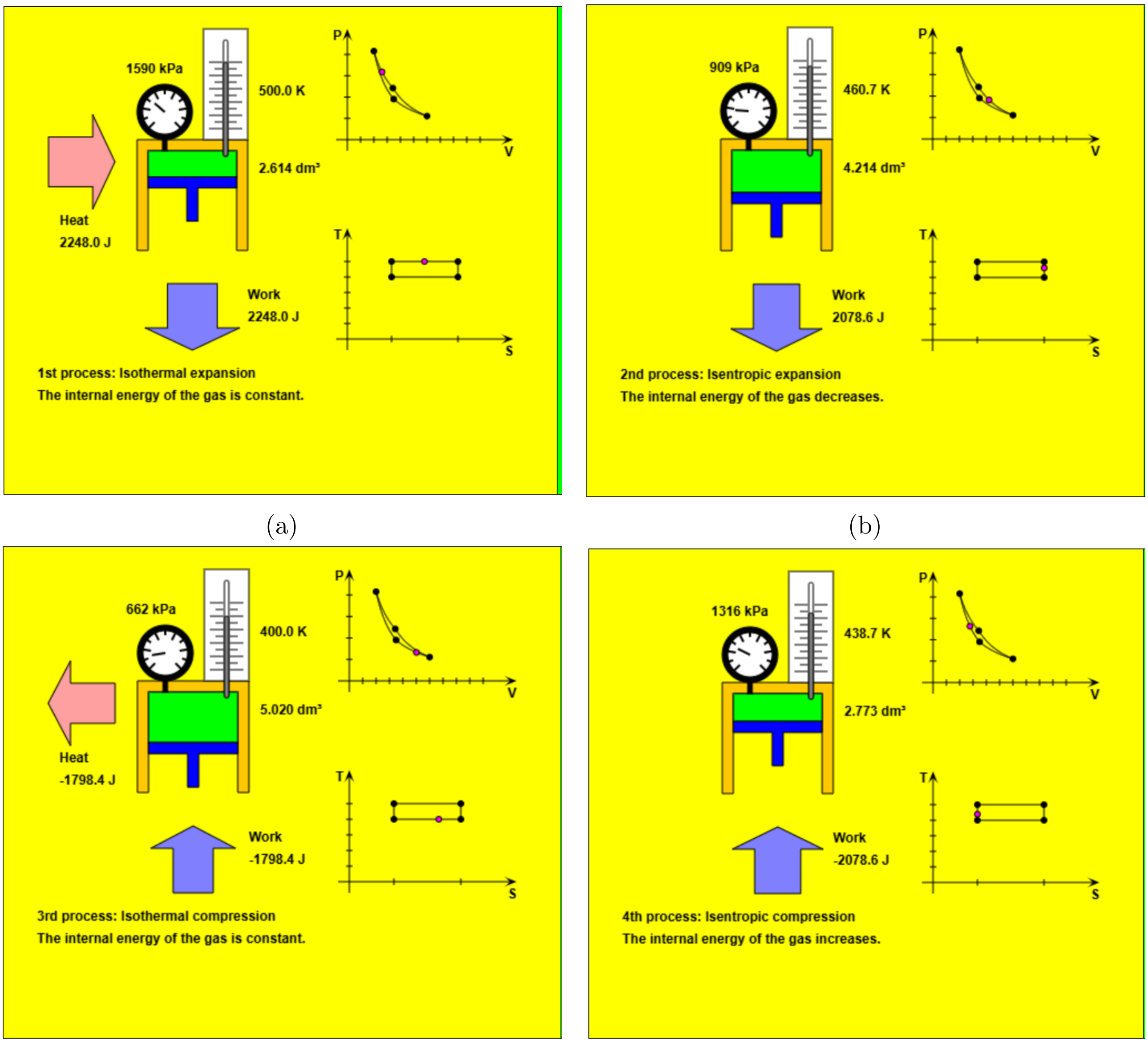
Isentropic expansion: The system doesn't accept heat from the source. Work is done due to expansion, decreasing the temperature and internal energy of the gas. This process continues till the temperature of the gas decrease to the temperature of the sink, that is, 400 K.

Isothermal compression: The gas is compressed at a constant temperature. It is in contact with the sink. Work is done on the gas to compress it and the heat is released to the sink as seen in the figure 3.

Isentropic compression: The gas is separated from sink, the process is adiabatic. The internal energy of the gas increases and the system is brought back to the initial state.

Process	P-V Relation	Work (in J)	Heat (in J)	Entropy Change (in $\frac{J}{K}$)
Isothermal expansion	$PV = K$	2248.0	2248.0	4.50
Isentropic expansion	$PV^\gamma = K$	2078.6	-	-
Isothermal compression	$PV = K$	-1798.4	-1798.4	-4.50
Isentropic compression	$PV^\gamma = K$	-2078.6	-	-

Table 1: Quantitative description of the processes. Positive work done shows work done by the system. Positive heat shows heat taken in by the system.

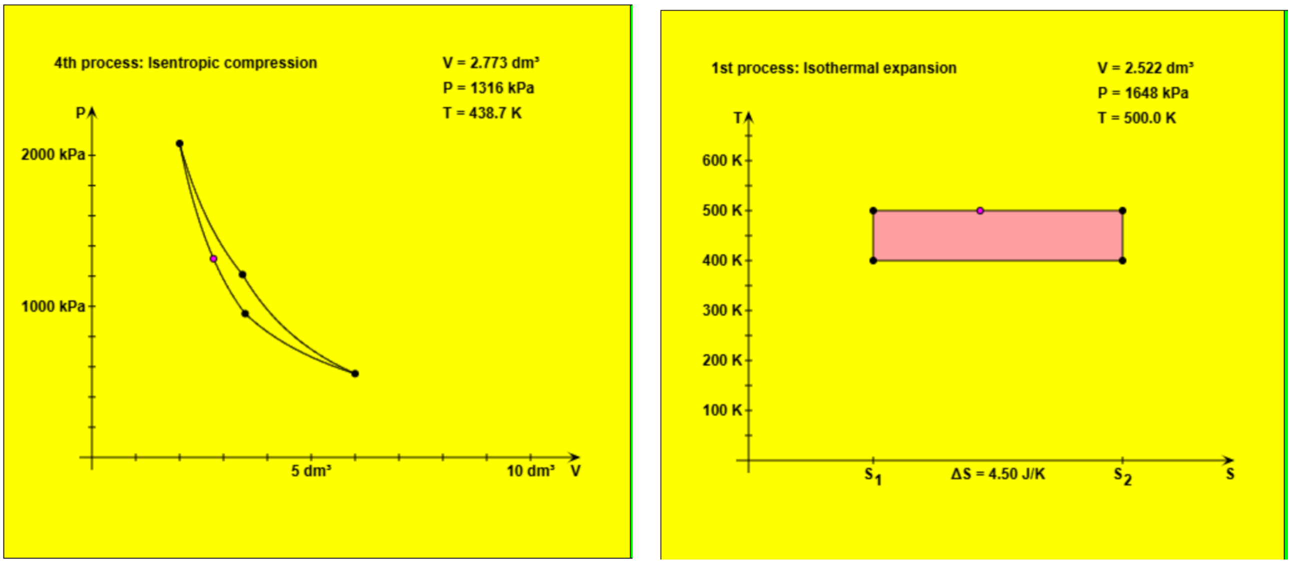


(a) (b) (c) (d)

Figure 3: The figure shows the 4-Stroke Carnot engine along with the flow of heat and work. (a) Isothermal expansion, (b) Adiabatic expansion, (c) Isothermal compression, (d) Adiabatic compression

4.2 Diagrammatic Representation

The four thermodynamic processes constituting the Carnot cycle can be also understood from the following two plots, Pressure-Volume and Temperature-Entropy graph. The app provides both graphs and we have plotted it for the same initial parameters.



(a) (b)

Figure 4: (a) The Pressure-Volume graph of the Carnot cycle. The area of the graph gives the total work done in the cycle. (b) Temperature-Entropy plot showing the isothermal and isentropic process.

From the P-V diagram we can see the expansion and compression as the increase and decrease of the volume respectively in X axis. In the isothermal case $P \propto V^{-1}$, and for adiabatic case (with $\gamma = 1.40$) $PV^{\gamma} = K$, a constant. Thus we see the following path in figure 4.[4]
In the T-S diagram we see constant entropy for isentropic process and change in entropy at a constant temperature during heat exchange. During the isothermal expansion process there is increase in heat at a constant temperature increasing the entropy and vice-versa.

4.3 Efficiency of the Engine

The efficiency of the Carnot engine can be easily calculated from the following eq 6. Given as,

$$\eta = 1 - \frac{T_L}{T_H}$$

In our experiment, we have specified $T_L = 400\ K$ and $T_H = 500\ K$. Thus, efficiency can be calculated as,

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{500} = 0.2$$

Percentage efficiency = 20%, which is an acceptable value for a real engine with mechanical friction and irreversibility. Efficiency of a real engine can never be equal to the efficiency of Carnot engine†.

†Ideal Carnot engine has unit efficiency. This is because we assume the heat reservoirs to be at constant temperature and all the processes to be reversible[1]

5 Conclusions

This demonstration of the Carnot cycle gives us an insight into the thermodynamics of a cyclic process. The ideal Carnot engine has unit efficiency and no other engine can have efficiency greater than it. The cycle is completed by four processes and the system returns to the initial state. The simulation gives a clear picture of steps and the details of heat and work. The engine we have worked with has a efficiency of 20 % and a diatomic gas as the working substance.

The interface we have used here Apps on Physics[2] provides the user with abundant materials to better understand physics concepts and apply it. The *Carnot Cycle* app has six inputs to modify the engine and control it manually. The diagrams give a better look at the broader picture and the plots give us the details of each step. This app can be further used with different gases and different working temperature of source and sink to simulate the changes due to it.

References

[1] H.B. Callen. *Thermodynamics and an Introduction to Thermostatistics*. Wiley, 1991.

[2] Walter Fendt. Apps on Physics.

[3] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. Fundamentals of Physics. Wiley, 2013.

[4] DC Pandey. *Understanding Physics for JEE Main and Advanced Waves and Thermodynamics*. Arihant Publication India Limited, 2021.