Combination of Resistors, Inductors, and Capacitors

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Apps on Physics by Walter Fendt[2]

Abstract
Connection of resistor, capacitor, and inductors in series and parallel are tricky and it's easy to make mistakes. The grouping of the components can be differentiated by applying the basic law of conservation of charge. From which we have derived that in series the current remains constant and in parallel the voltage remains constant. Using the Apps on Physics we have simulated each case and solved a simple problem of mixed connections to show an application of the app.

1 Aim of the experiment

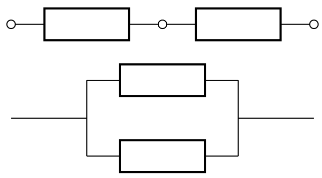
To understand the connection of circuit elements using the **Apps on Physics**. The following applications will be demonstrated:

1. Understanding the series and parallel connection
2. Impedance, phase angle, power factor etc. in RLC circuits
3. Use the app to solve circuit analysis problems

2 Introduction

2.1 Series and Parallel Connection in a Circuit

Any two terminal device with a voltage-current relation defined by some mathematical formula is called a circuit element. Resistor, capacitor, and inductors are passive circuit elements that do not produce energy. They can be connected either in **series** or **parallel** in the circuit.



The series and parallel connection are shown above respectively. In series connection, the current across the connection does not drop, the voltage is divided according to some rule. In parallel connection, the voltage drop is equal to the source voltage V across the connection and the current is divided.[4]For each circuit element the equivalent value can be found out using specific formulae. We have gone through mixed connections in the experiment and the have shown the various properties for the different elements.

2.2 Concepts in RLC Circuit

When resistance, inductance, and capacitance are present in a circuit with an AC input, it's called a RLC circuit. In case of series R-L circuit there is inductance present. Hence the voltage leads the current by a phase of $\frac{\pi}{2}$. In series R-C circuit, due to capacitance, the current leads voltage by a phase angle of $\frac{\pi}{2}$.
In case of series RLC circuit, the total resistance to the flow of current is called *impedance*. The phase angle (ϕ) and impedance (Z) are given by,

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \tag{1}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{2}$$

where, $R =$ Resistance
$X_L = \omega L$ (Inductive Reactance)
$X_C = \frac{1}{\omega C}$ (Capacitive Reactance) In RLC circuit, the total energy from the AC input is stored in fields as well as dissipated by the resistance.[3] Thus, the average power supplied is given by,

$$P = V_{\text{rms}} I_{\text{rms}} \cos\phi \tag{3}$$

where ϕ is the same phase we have previously discussed. For maximum transfer of power, we want to minimize the phase.

2.3 Circuit Analysis

Circuit analysis comprises of various methods like Kirchhoff's laws, Nodal analysis, the Superposition theorem, Thévenin's and Norton's theorems, star/delta transformation and the maximum power transfer theorem. The main aim is to find current or voltage through specific elements.
We can find current through a circuit element by connecting an **ammeter** in series with it. To measure the voltage across a circuit element, we connect a **voltmeter** in parallel to the element.
We have taken a simple circuit in *Example 14.5* from *Electrical and Electronic Technology*[1] and tried to simulate the circuit using the app and match the analytical solution provided in the book.

3 Methodology

3.1 Circuit Formation and Initialization

We have utilized the *Apps on Physics*[2] to simulate the required circuits and solve problems. First, the interface is simple as seen in figure 1 having an input AC with variable voltage and frequency. In the right-hand side we have options to add/replace circuit elements with specific values in series or parallel. The brown rectangle shows the currently selected component and the new components are added with respect to it. Now to measure current or voltage through a particular circuit element, we have options to add Ammeter or Voltmeter at the bottom right corner.

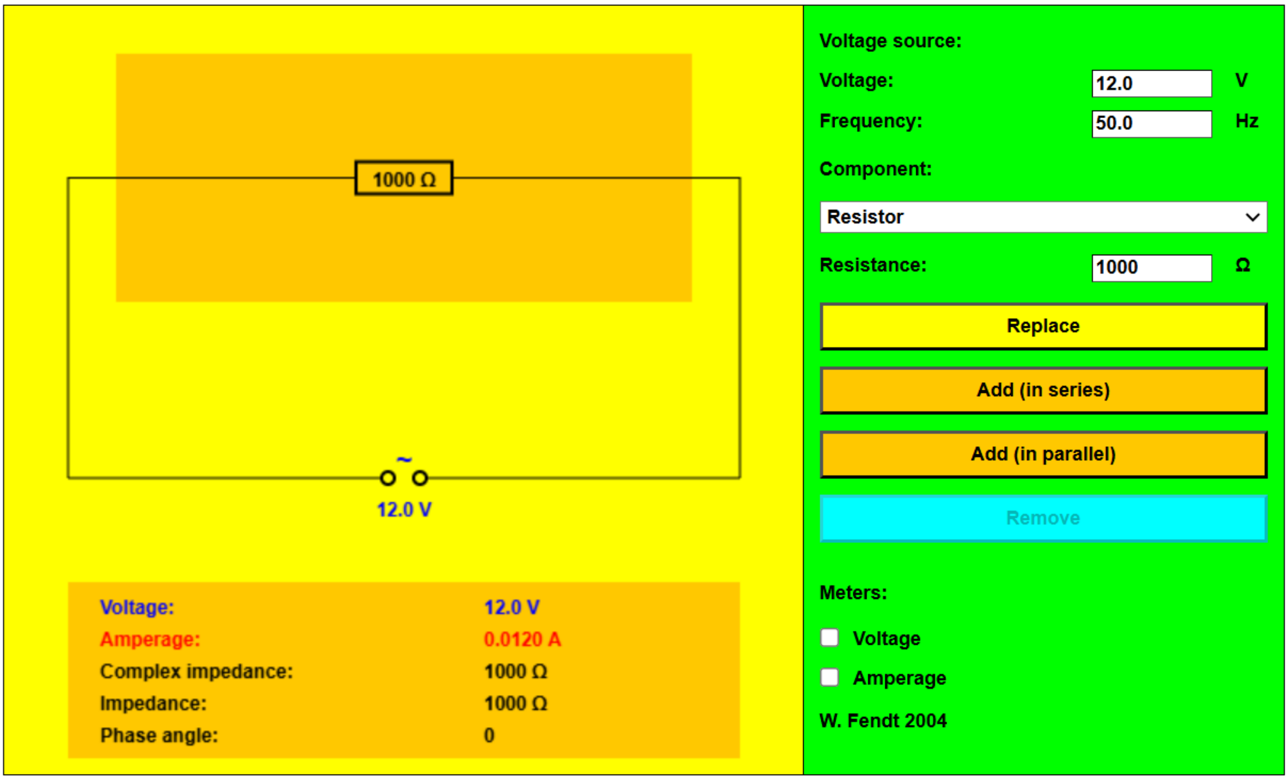


Figure 1: Interface of the *Combinations of Resistors, Inductors and Capacitors* app.

Components	Range
Resistor	$1 - 10^6\Omega$
Capacitor	$10^{-4} - 10^3\mu f$
Inductor	$10^{-4} - 10^3H$

Table 1: Range of the input circuit elements.

However, we can not attach any arbitrary value to the components. They have a range of values that can be allotted, as we have shown in table 1 for resistor, capacitor, and inductor.

Now, we can use any values present in the range as shown above. We have created the circuit given in *Example 14.5* [1] and measured the respective values.

3.2 Simulation and Data collection

Creating a circuit is easy and can be done by anyone. Initially, the interface comes with an AC input and a resistance. We have added more elements to it by using the options given on the right side. A total amount of **3** elements can be added either in series or parallel in this app.

We have used different values of each element in both series and parallel to verify the connection formulae derived from the inherent physics of these components. Also, we have constructed the circuit given in the problem to find out the current through the capacitance and voltage across the terminals of the resistor.

For the user to utilize this app in finding the current or voltage in the circuit, the options at the bottom right of figure 1 can be used. We have showcased a simple case of this connection

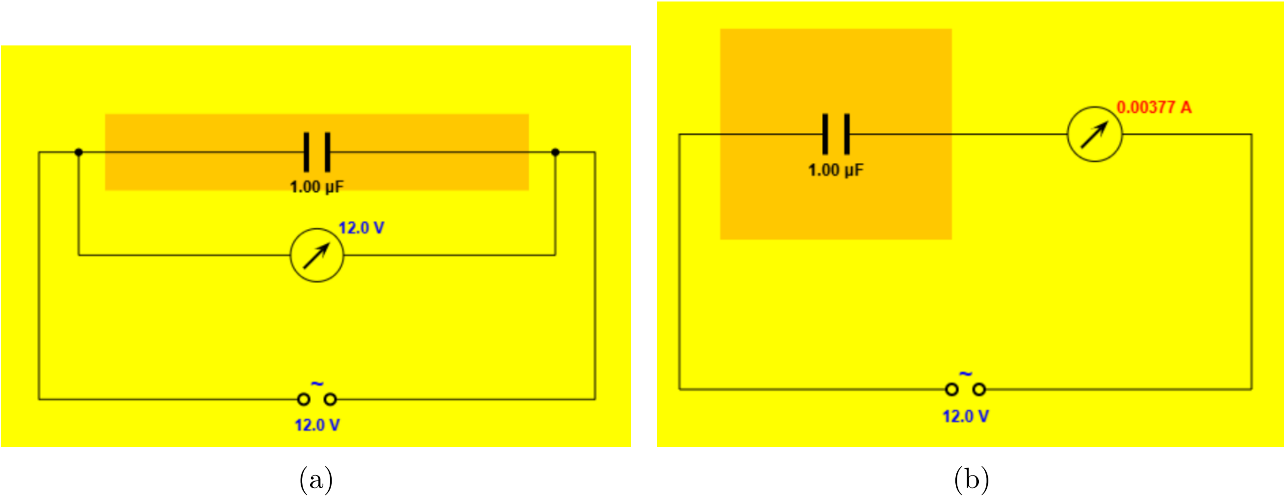


Figure 2: Method to find the voltage and current through a circuit element. (a) Voltmeter connected in parallel to the capacitor shows the voltage across the two terminals. (b) Ammeter connected in series shows the current flowing through the capacitor.

in figure 2 where first the circuit element is selected.* After selecting it both ammeter and voltmeter can be connected to it. The method of connection is taken care by the system.

4 Results

4.1 Series and Parallel Connection

The data collected by changing the resistance, capacitance, and inductance in both series and parallel connection, we verified the theoretical formulae for effective value. In case of resistance, in series current I remains the same throughout the circuit. Thus,

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ IR_{eq} &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \end{aligned}$$

Now to find the equivalent resistance:

$$R_{eq} = R_1 + R_2 + R_3 \tag{4}$$

Similarly, in parallel connection the voltage V across each element remains constant. Hence we can write,

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ \frac{V}{R_{eq}} &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ &= V(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}) \end{aligned}$$

*When the element is selected it is highlighted with a brown rectangle

Hence, the equivalent resistance for parallel connection is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{5}$$

Similar to equation 4 and 5 we can tabulate all the formulas for the connections in case of capacitor and inductor.
We have used components with $R = 1000\Omega$, $C = 1.00\mu f$, and $L = 1.00H$ for our demonstration.

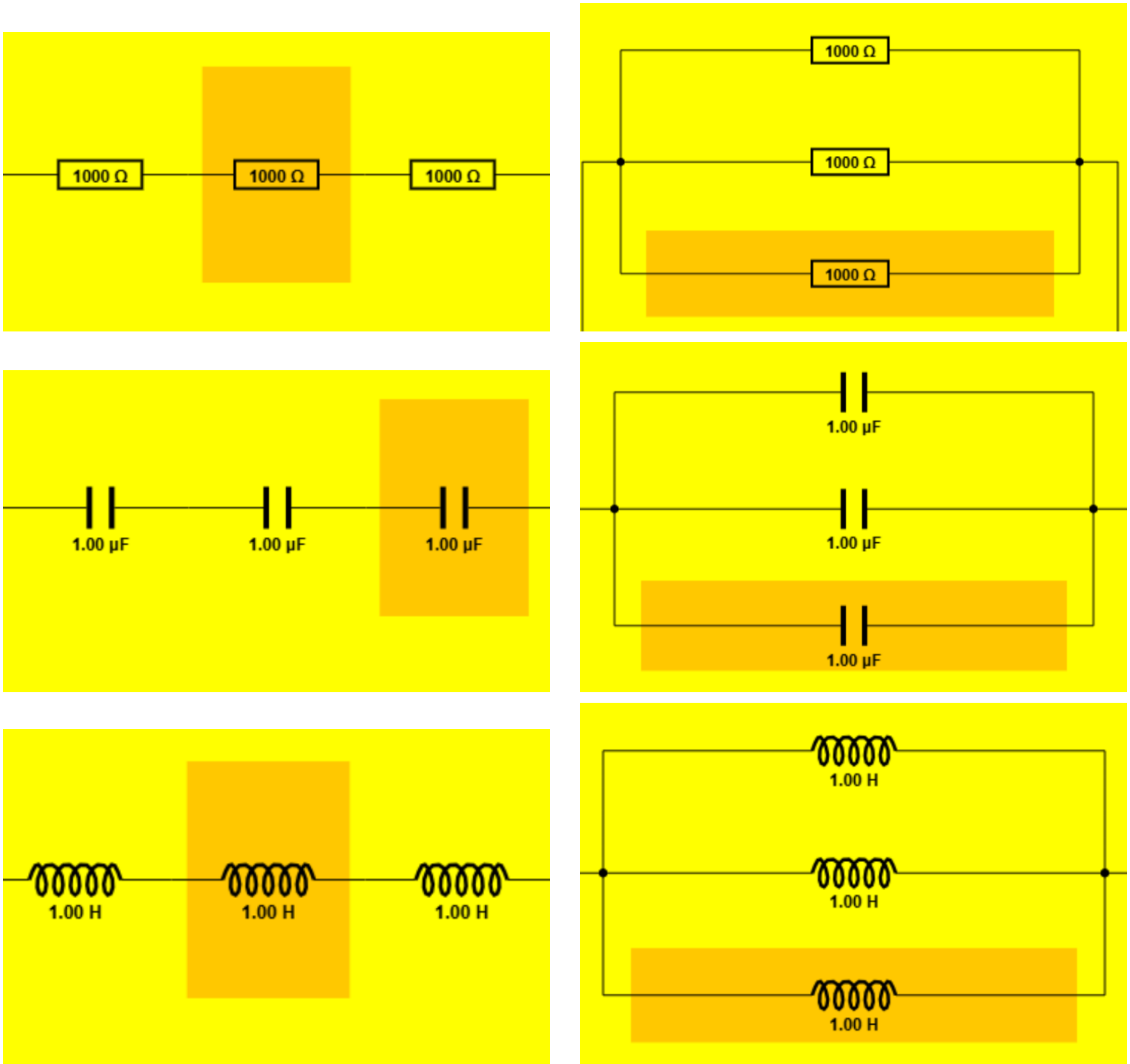


Figure 3: Series (left) and Parallel (right) connection of resistor, capacitor, and inductor from top to bottom.

As we can see in the figure 3 the series connections are in the left and the parallel connections are shown in the right hand side.

Now taking the defined values of the components we have tabulated the formulae of connection along with the equivalent values in both series and parallel connections. The equivalent values are taken from the simulation and verified by the formulae. Table 2 shows the results:

Components	Series	Equivalent	Parallel	Equivalent
Resistor	$R_{eq} = R_1 + R_2 + R_3$	3000 Ω	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$	333 Ω
Capacitor	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$	0.33 μf	$C_{eq} = C_1 + C_2 + C_3$	3 μf
Inductor	$L_{eq} = L_1 + L_2 + L_3$	3 H	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$	0.33 H

Table 2: Results of the formulae for connection of components and the equivalent values calculated from the app.

To find the equivalent value of capacitance in series connection, the app shows the impedance $Z = 9549\Omega$. Now for only capacitors present in the circuit $Z = X_C$,

$$\begin{aligned} X_C &= \frac{1}{\omega C_{eq}} \\ &= \frac{1}{2\pi f C_{eq}} \\ \implies C_{eq} &= \frac{1}{2\pi f X_C} \\ \implies C_{eq} &= \frac{1}{2\pi \times 50 \times 9549} \\ \implies C_{eq} &= 0.33 \times 10^6 = 0.33\mu f \end{aligned} \tag{6}$$

Similarly we have done for the other cases using the formulae given in table 2 and verified with the data produced in the app to fill out the equivalent values.

4.2 Solving the Circuit Analysis Problem

In this section, we have considered the circuit shown in the figure 4. Let us define the problem once again,

1. Find the current, voltage, and impedance for each element in the circuit for the input of $2V$, $1000Hz$.
2. Find the total impedance and phase angle of the circuit for a given input of $1V$, $50Hz$.

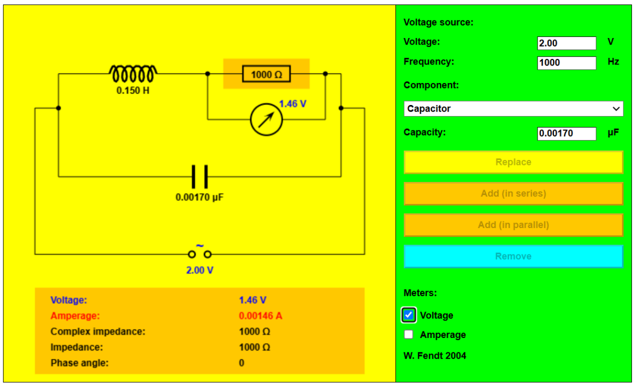
The results of the first problem can be seen in the table:

Components	Voltage (in V)	Current (in A)	Impedance (in Ω)
Resistor	1.46	0.00146	1000
Capacitor	2.00	0.0000214	93621
Inductor	1.37	0.00146	942

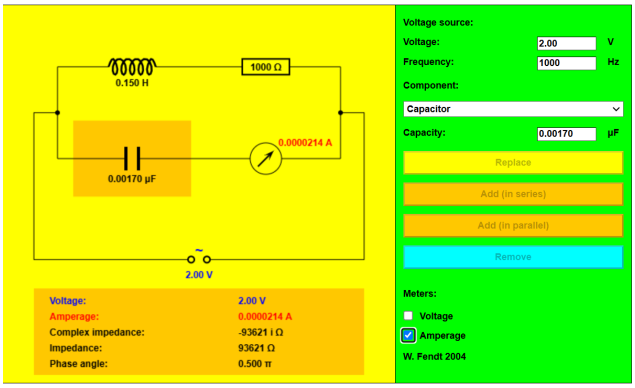
Table 3: Experimental values obtained from the simulation.

Now to find the solution to the second problem figure 4 (c) is used. From the data we can see

- The total impedance of the circuit for the given input is $Z = 1001\Omega$.
- The phase angle $\phi = -0.0148\pi$.



(a)



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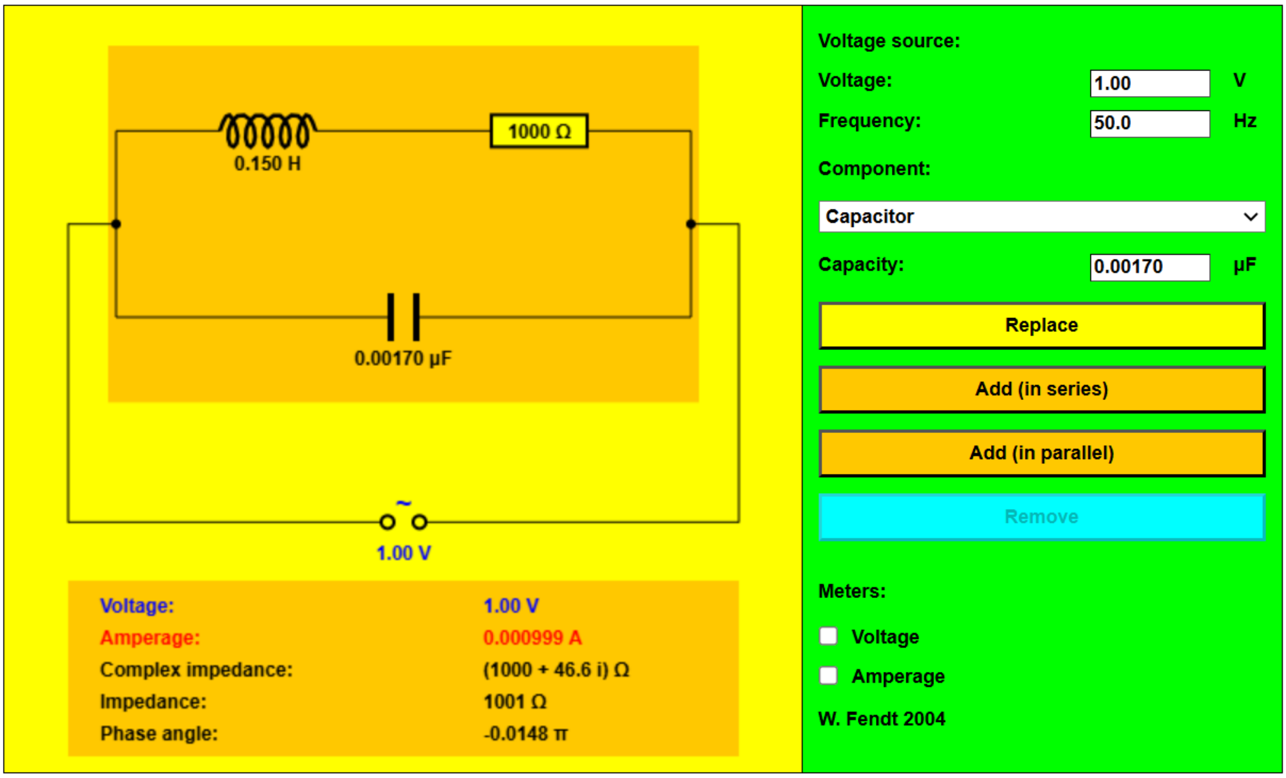

(c)

Figure 4: The figure shows the simulated circuit for the given problem. It gives us an idea of the methods used to find the results required to solving the problem. (a) Voltmeter is connected in parallel to the resistor to find the voltage drop across it. (b) Ammeter is connected in series with the capacitor to measure the current flowing through it. (c) The total circuit is selected for a given input. It shows the required data related to the circuit at bottom left corner.

5 Conclusions

In a simple circuit with only resistance, we do not face much problem in adding other resistances in series or parallel. But when other components are present, they do not follow the algebraic addition rules as resistance. There is also an additional phase between the voltage and current. Thus in these kind of situations we use the modified expressions and proper connection formulae to get our way around. The simple cases of series and parallel cases are demonstrated and the formulae are tabulated. At the end we have solved a problem of mixed connections from a standard book.

The Apps on Physics gives us a simple yet powerful tool to have a deeper understanding and visualization of these concepts. It is a great way to learn circuit connections and the intricate details that are associated with it.

References

[1] Hughes Edward. *Hughes electrical and electronic technology.* Pearson Education India, 2010.

[2] Walter Fendt. Apps on Physics.

[3] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics.* Fundamentals of Physics. Wiley, 2013.

[4] D. Pandey. *Understanding Physics for JEE Main and Advanced Electricity and Magnetism.* Arihant Publication India Limited, 2022.