Electromagnetic Oscillating Circuits

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Apps on Physics by Walter Fendt[2]

Abstract
Electromagnetic Oscillating Circuits app is a great tool to understand the LC oscillation, RLC damping, etc. for students. Our report tries to shed some light on three important aspects of electric oscillation using the app. The simulation gives correct values for the frequency of oscillation as verified theoretically. The animations of V-I plot and the transfer of energy for LC and RLC circuits provide visual insight into the internal physics of the phenomenon. This type of simulation can be utilized by intermediate students to have a practical knowledge and experience in the above topic and have a better grasp of the theory behind it.

1 Aim of the experiment

To understand the following concepts using **Apps on Physics**:

1. Frequency of oscillation
2. V-I plot and Phase
3. Conservation of energy in LC oscillation

2 Introduction

2.1 Oscillation in nature

Particles in nature start to oscillate whenever they are slightly displaced from their stable equilibrium position. The restoring force that acts on the particle to bring it back to the minimum potential energy drives the motion. The simplest cases where motion can be described by sinusoidal waves are called simple harmonic motion. The mass in a spring of force constant k is one of the most fundamental simple harmonic oscillations seen in nature. Similar thing happens in an electromagnetic circuit consisting of a capacitor and an inductor. If a charged capacitor C is short-circuited through an inductor L,the charge and current in the circuit start to oscillate simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. With these idealizations, the oscillation in the circuit continues indefinitely, and the energy is periodically transferred from the electric fields of the capacitor to the magnetic fields of the inductor keeping the total energy constant. A crude analogy can be drawn between the spring-mass system and LC oscillation considering the capacitor and inductor acting as spring and mass respectively.[5]

2.2 LC circuits and Frequency of Oscillation

Inductor and a charged capacitor connected in a circuit produce electromagnetic waves due to oscillation of charge. The equation of motion can be written as

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \tag{1}$$

with R/L acting as the damping factor. In an ideal circuit $R = 0$, and the motion takes place indefinitely. But in presence of resistance, it slowly dies down losing energy due to damping. The solution of this second order ODE with $R = 0$ is given by

$$q(t) = q_0 \cos(\omega t) \tag{2}$$

Where the angular frequency of the oscillation[1] is given by:

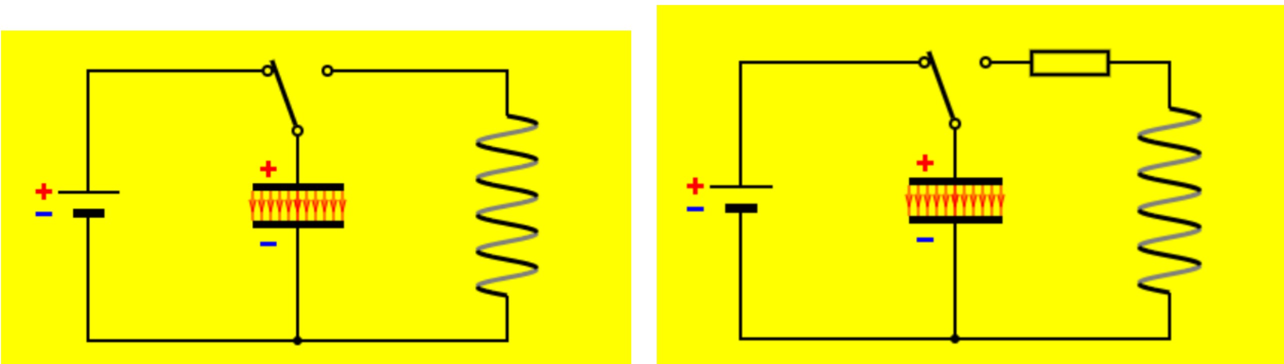
$$\omega = \frac{1}{\sqrt{LC}} \tag{3}$$

2.3 Conservation of energy and dissipation

There occurs an interplay between the energy of static charge (Electric field energy) and energy of the moving charge (Magnetic field energy) in each cycle of oscillation. Initially, when the capacitor is fully charged, the total energy is stored completely in the electric fields of the capacitor. When the capacitor gradually discharges, the charge moves through the inductor and energy is stored in the magnetic fields. The total energy is conserved.[3] But, in practice there is finite resistance in a circuit, which causes the dissipation of energy. The energy is released as heating of the resistance and the oscillation dies down gradually.

3 Methodology

3.1 Circuit creation



(a) LC Circuit

(b) RLC Circuit

Figure 1: LC and RLC circuit.

The Electromagnetic oscillating circuits app can be found in the website Apps on Physics[2].The Apps on Physics is used to add the circuit components with the corresponding values. From the

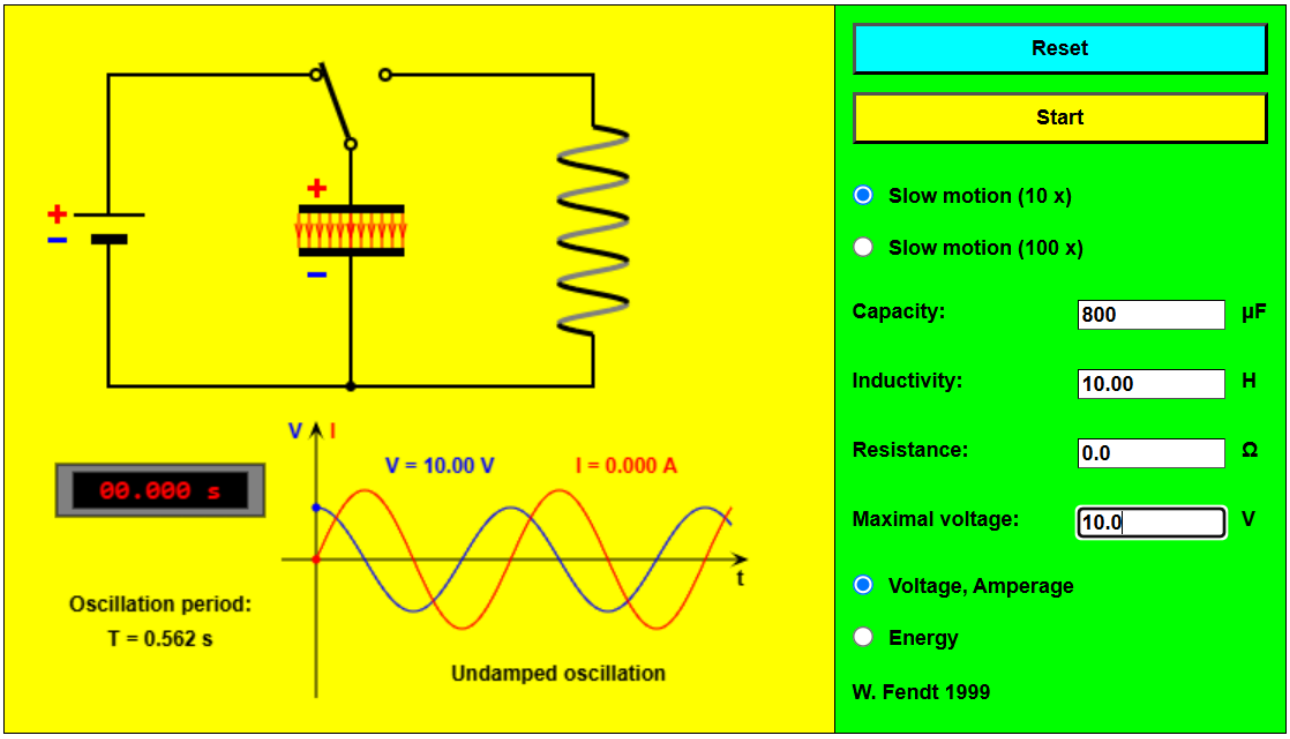


Figure 2: Interface of the application. Various parameters can be seen on the right hand side. The frequency of oscillation is on the bottom left corner. The V-I characteristic plot is in the center.

application we have used both LC and RLC circuit to understand ideal and actual cases of an oscillation respectively. For LC circuit we use: Input $V = 10.0V, L = 10.00H$, and $C = 800\mu f$. For the RLC circuit we added a resistance $R = 50\Omega$. After setting up the circuit we clicked on start to close the key and complete the circuit. The final circuits can be seen in the figure 1.

3.2 Circuit simulation

We go to the app and set-up the circuit elements to their desired values given in the previous section. The interface allows us to find the time period of oscillation, V-I plot, and the energy transfer in the app. The animation can also be slowed down by 100x using the option given in the right. We have utilized these functions to find the various parameters related to our study and compared them with the results from analytic methods. The interface of the app is shown in the figure 2. The clock timer on the left side shows the time for which the simulation is being carried out. It is used to make a rough idea of the time taken for the oscillation in the RLC circuit to die down.

4 Results

4.1 Frequency of oscillation

The oscillation period of the LC circuit with the given values of inductance and capacitance is found out to be $T = 0.562\,\mathrm{s}$ from the above figure 2. To find the angular frequency of the oscillation ω we use the formula:

$$\omega = \frac{2\pi}{T} \tag{4}$$

Putting in the values for T we find,

$$\omega = \frac{2 \times \pi}{0.562} = 11.18\,\mathrm{s}^{-1}$$

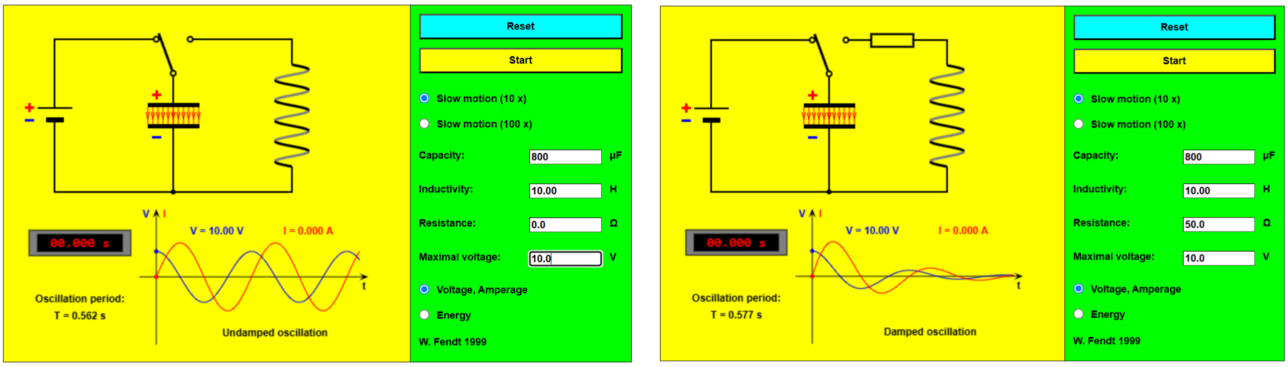
Approaching this from a theoretical perspective, the analytical solution for ω can be found from the equation 3 where we put $L = 10.00H$ and $C = 8.0 \times 10^{-4}\mu f$. The angular frequency is calculated to be:

$$\omega = \frac{1}{\sqrt{10 \times 8.0 \times 10^{-4}}} = 11.18\,\mathrm{s}^{-1}$$

We can see the values match perfectly and the simulation is helpful in finding the time period of oscillation for any set values of RLC. The formula for the calculation of angular frequency for non zero resistance is a bit complicated and we will not discuss it here. But the app finds it out using the internal calculation methods for values set by the user. The values $R(0\Omega - 1000\Omega)$, $C(100\mu f - 1000\mu f)$, and $L(1H - 10H)$ are the lower and upper limits.

4.2 Voltage-Current (V-I) plot

Initially, when the capacitor is fully charged, the total energy is stored in its electric field. At this point, the current is zero, and the voltage across the capacitor equals the input voltage, $V = 10$V. As the capacitor slowly discharges, charges move through the circuit and generate a magnetic field in the inductor. Consequently, the voltage across the capacitor decreases while the current in the circuit increases. At the quarter-cycle, as shown in Figure 3, the voltage becomes zero, and the current reaches its maximum value. This cycle repeats periodically with a time period of $T = 0.562$s. The electromagnetic oscillation continues indefinitely, provided there is no energy dissipation.



(a) (b)

Figure 3: V-I plot for two different circuit configuration.(a) Ideal LC circuit with sinusoidal oscillation propagating infinitely. (b) In the presence of resistance in the circuit, there is a decay in the amplitude of voltage and current in the plot.

Whenever there is resistance in the circuit, we see a decaying voltage and current amplitude in the V-I plot, figure 3(b). The oscillation still persists, but the magnitude of current and voltage becomes negligible after around 3 seconds of oscillation.

We see a delay of $\frac{\pi}{2}$ between the voltage and current in an ideal LC circuit. This delay between

V and I is called phase difference, and denoted by ϕ. The factor $\frac{\pi}{2}$ comes due to the initial charge in the capacitor, which makes the circuit purely capacitive. In a purely capacitive circuit, the current leads the voltage by $\frac{\pi}{2}$. For a more general case, the phase difference is given by[1]:

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \tag{5}$$

But for LC oscillations $R = 0$, hence the phase is always the same factor $\frac{\pi}{2}$. The following fact can be seen from the simulation.

4.3 Energy and its dissipation

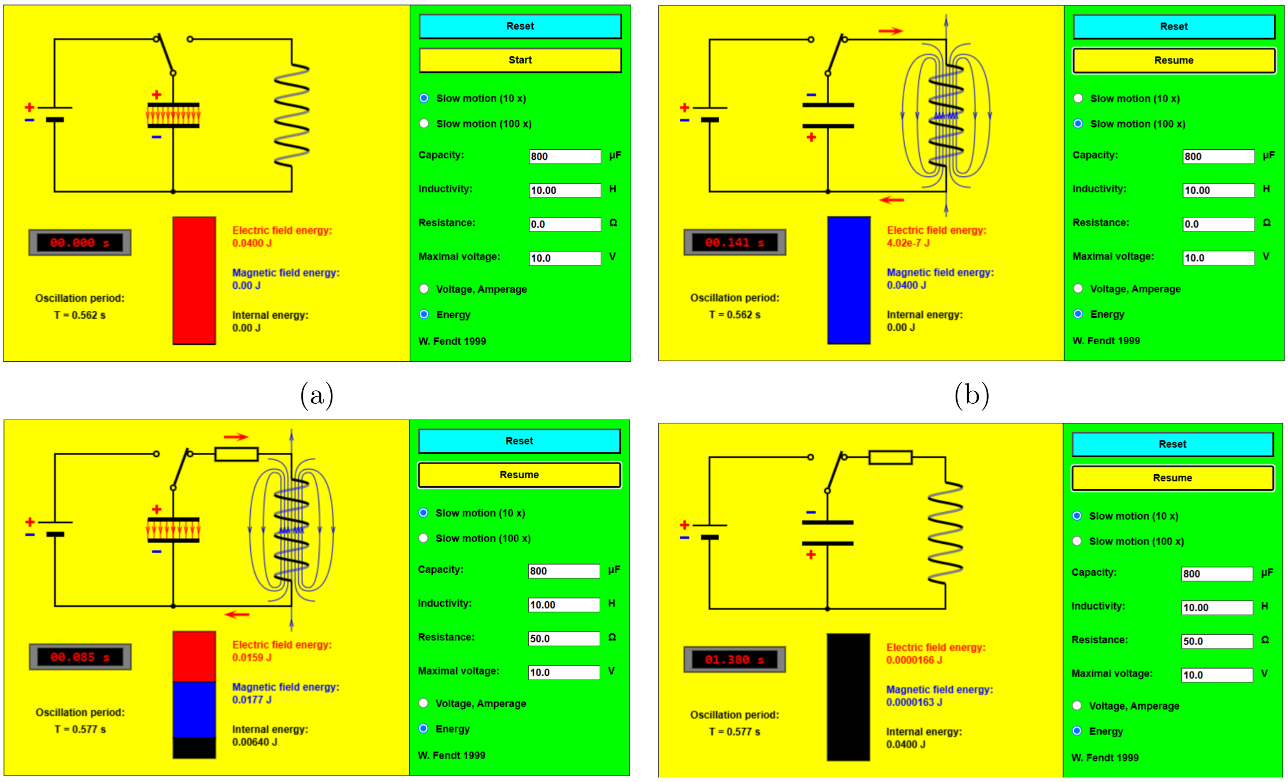


Figure 4: Energy distribution in different fields and its dissipation. (a) Initial charged capacitor with total energy stored as electric field (Red). (b) $0.400J$ energy now converted to magnetic field and stored in the inductor (Blue). (c) RLC circuit showing energy dissipated by the resistor as black. (d) After a certain time almost all the energy is dissipated out and the oscillation damps.

Energy is provided by the input voltage by doing work to charge the capacitor. This energy is stored in the capacitor in the form of electric fields. Electromagnetic fields can store energy and momentum. This can be simply verified as microwave ovens produce EM waves that tend to heat the food.[4] In the case of our circuit, the presence of a resistance causes the energy to flow out of the system by Joule's heating, causing the oscillation to eventually dampen out.

In the above figure 4 we can see in the case of an LC circuit there are no dissipator of energy. Hence, the total energy remains constant in the system. But after adding resistor to the circuit, it heats up when current passes through it. This heating energy flows out of the system each cycle gradually decreasing the amplitude of V and I. The illustration in the figure 4 shows that in LC circuit total conversion from electric field to magnetic field takes place. But in the RLC circuit there is dissipation which was also visible in the V-I plot in figure 3.

Inductance(in H)	Capacitance(in μf)	Resistance(in Ω)	Time period(in s)	Oscillation type	Total energy(in J)
5	200	0	0.199	Undamped	0.0100
5	400	0	0.281	Undamped	0.0200
5	200	100	0.209	Damped	0.0100
10	200	100	0.288	Damped	0.0100
10	400	500	-	Overcritically Damped	0.0200

Table 1: Experimental data for different values of the input parameters.

In the the above Table 1, we have used different values of the circuit elements to find the changes in output parameters. For a fixed input voltage of 10 V we have tabulated the data derived from the app. From the above Table 1 we can draw the following inferences:

I. The total energy depends solely on the capacitance for a fixed V, as initially it is charged and stores the energy in the field.

II. The oscillation is undamped and continues forever when the resistance is zero.

III. In presence of resistance, energy dissipation occurs. The oscillation is damped and the time period increases.* (Compare row 1 and 3)

IV. High resistance causes overcritical damping which distorts the oscillation and it starts to look like an exponential decay. There is no concept of time period in this type of oscillation.

This simulation is helpful in visualizing the concept of energy and its dissipation easily by setting different values of capacitance, resistance, and inductance. Some limiting values and cases were shown in this experiment and are useful to understand the physics of oscillation.

5 Conclusions

Here in this experiment we have produced and verified the calculations of angular frequency using analytical method. The results are accurate with the values derived from the app. The V-I characteristics of the oscillation are plotted for a specific value of R, L, and C. There exists damping in the presence of resistance where the energy of the system is not conserved, and the energy is dissipated as internal energy through the resistance. The transfer of energy is properly mapped in the app with different colors. Data collection with different values of parameters give us insight into the physics of oscillation.
Apps on Physics[2] provide students with abundant materials to better understand physics concepts practically. The electromagnetic oscillating circuits app offers users a simulation of

*The change in time period is due to the different formula used to calculate the frequency in presence of resistance. We have not discussed it here due to its complexity circuits and the ability to manually adjust the values of circuit elements. However, due to its simplicity, the app does not support rigorous calculations of various parameters. Nonetheless, the V-I plot and energy-transfer visualization enable students to comprehend the internal workings of circuits and concepts such as conservation of energy, phase, and damping.

References

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