Visualizing Linear Programming

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Introduction

Linear programming is a powerful mathematical technique used to optimize a specific objective, such as maximizing profit or minimizing cost, subject to certain constraints. It has practical applications in industries such as manufacturing, logistics, and finance. In this project, we demonstrate how to solve a linear programming problem using GeoGebra, a dynamic mathematics software. This project focuses on optimizing the profit of a company producing two products under labor constraints. The visualization capabilities of GeoGebra are utilized to make the solution process intuitive and interactive.

Methodology

The solution involves the following key components:

- **Objective Function**: A mathematical representation of the goal to optimize (maximize or minimize).
- **Constraints**: A set of linear inequalities representing limitations on resources.
- **Feasible Region**: The graphical intersection of all constraints, representing possible solutions.
- **Optimization**: Identifying the vertex of the feasible region that gives the optimal value of the objective function.

GeoGebra is used to:

- Graph the constraints.
- Highlight the feasible region.
- Visualize the objective function and find the optimal solution interactively.

Procedure

The steps to solve the problem in GeoGebra are as follows:

1. Define the Problem

- **Objective**: Maximize profit $Z = 40x + 50y$, where x and y represent the units of products A and B, respectively.
- **Constraints**:
  - $2x + 3y \leq 60$ (labor constraint)
  - $x \geq 0$ and $y \geq 0$ (non-negativity constraints)

2. Set Up GeoGebra

Open the **Graphing Calculator** in GeoGebra.

3. Input Constraints

Plot the constraints as inequalities:

- $2x + 3y \leq 60$
- $x \geq 0$
- $y \geq 0$

The shaded intersection represents the feasible region.

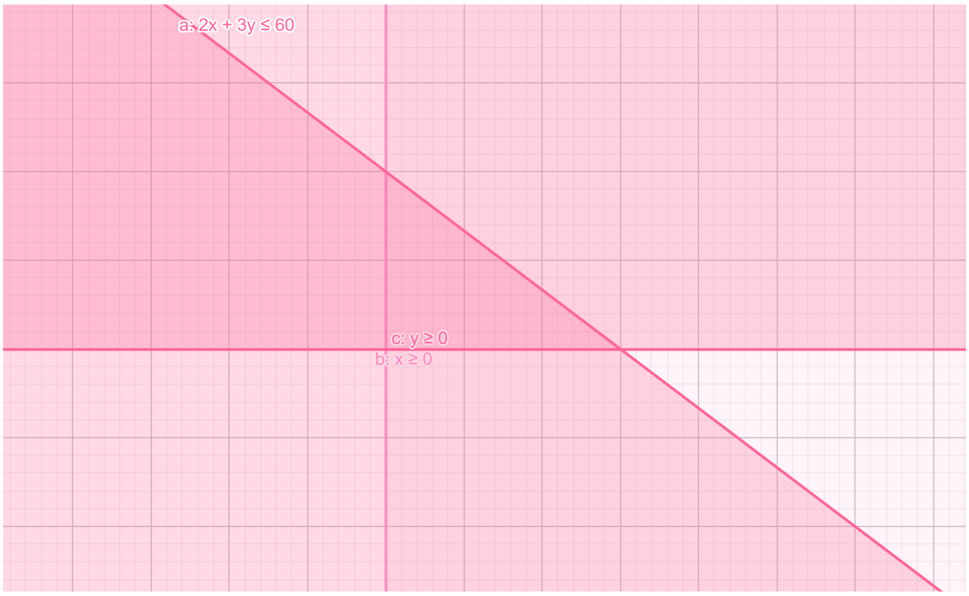

Figure 1: Constraints plotted

4. Identify the Feasible Region

Determine the vertices of the feasible region by finding intersections of the constraint lines:

- $(0, 0)$, $(0, 20)$, and $(30, 0)$.

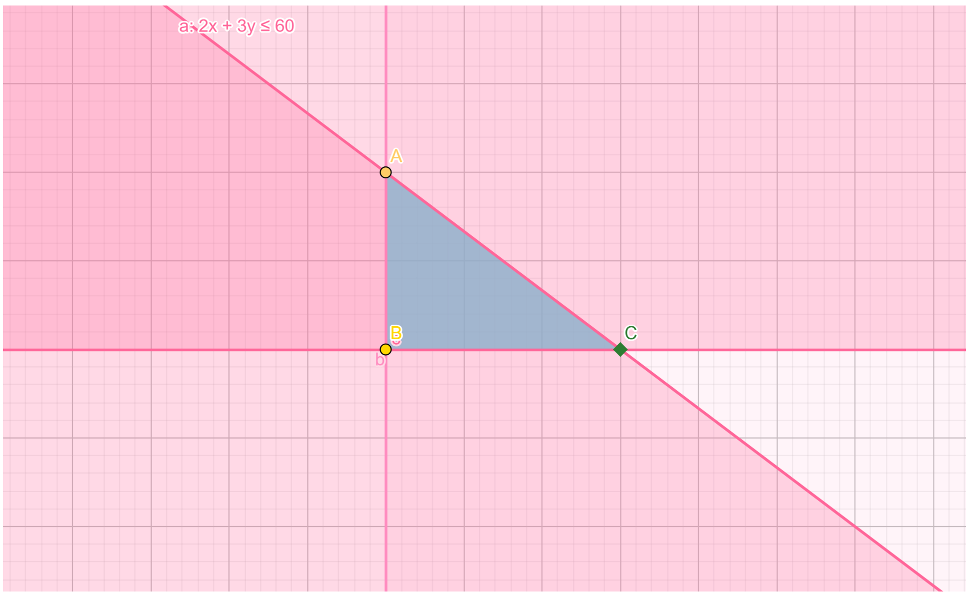

Figure 2: Vertices of the feasible region forming a triangle

5. Enter the Objective Function

Write the profit function $Z = 40x + 50y$. Use a slider to visualize how $Z = k$ changes as the line moves through the feasible region.

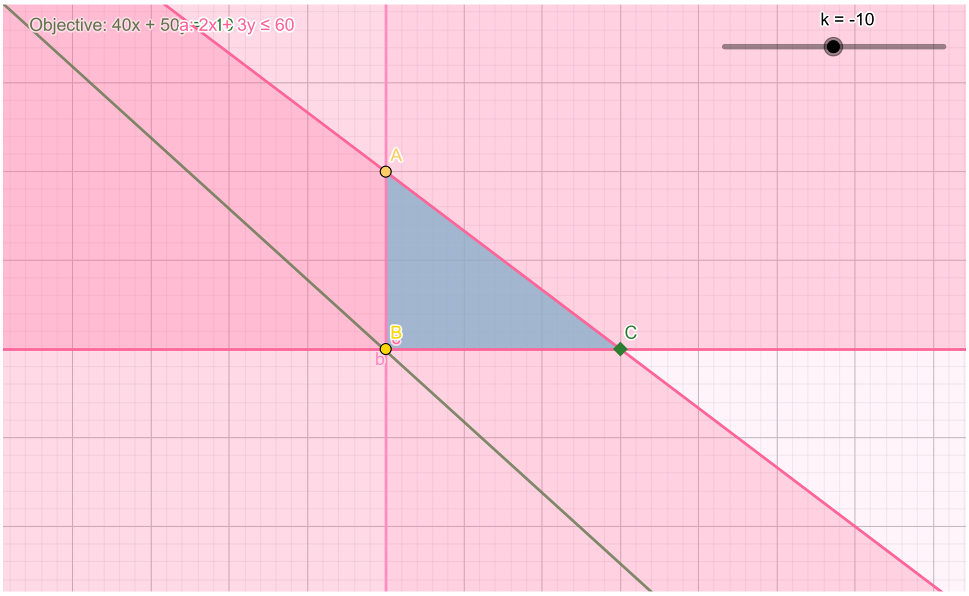

Figure 3: Profit function visualization with slider

6. Evaluate at Vertices

Compute Z for each vertex:

- $Z(0, 0) = 0$
- $Z(0, 20) = 1000$
- $Z(30, 0) = 1200$

The maximum profit $Z = 1200$ occurs at $(30, 0)$.

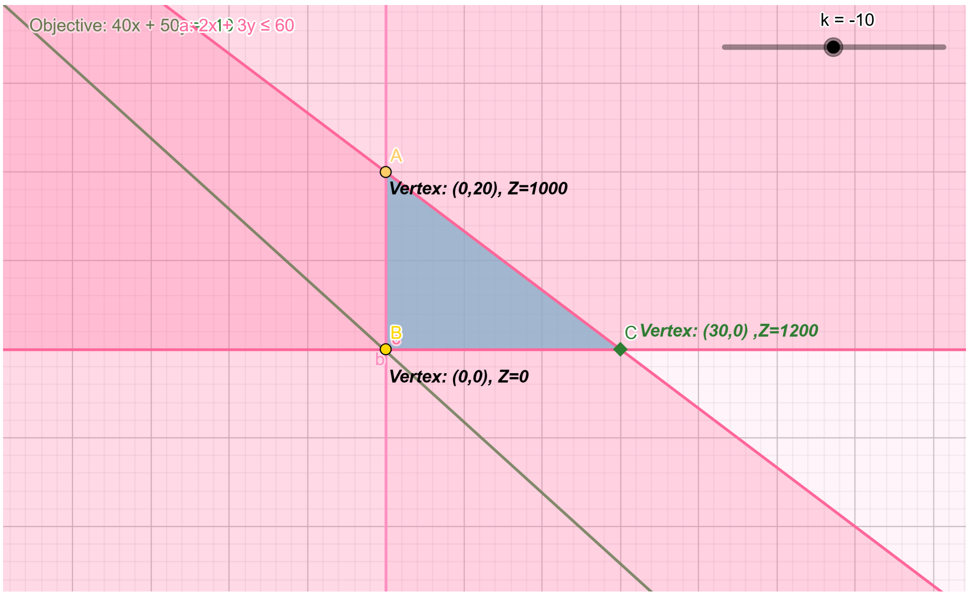

Figure 4: Evaluating Vertices

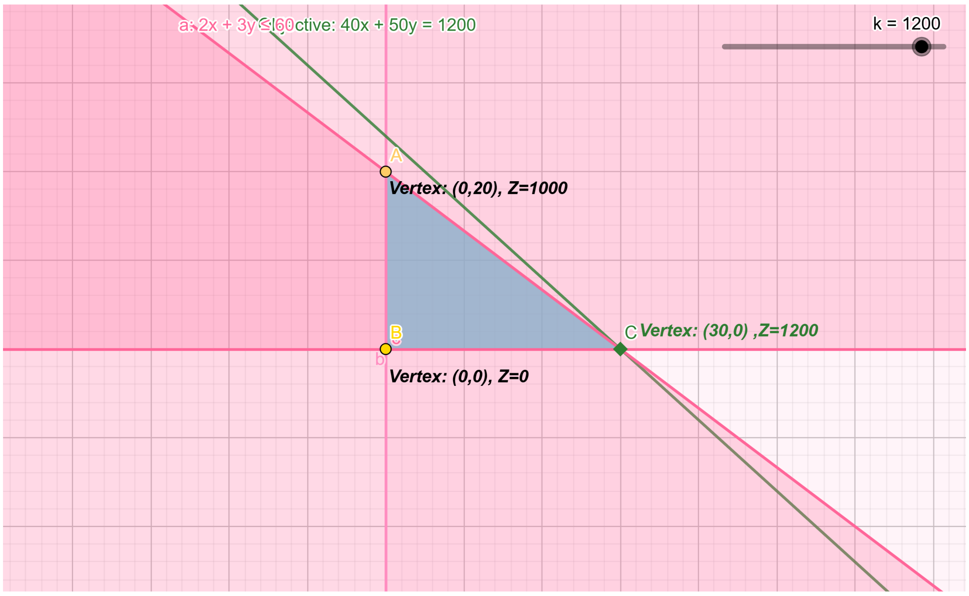

Figure 5: Profit Function Visualization

7. Highlight and Annotate

Shade the feasible region. Label the constraints, vertices, and the optimal solution. Add a legend to explain the graphical elements.

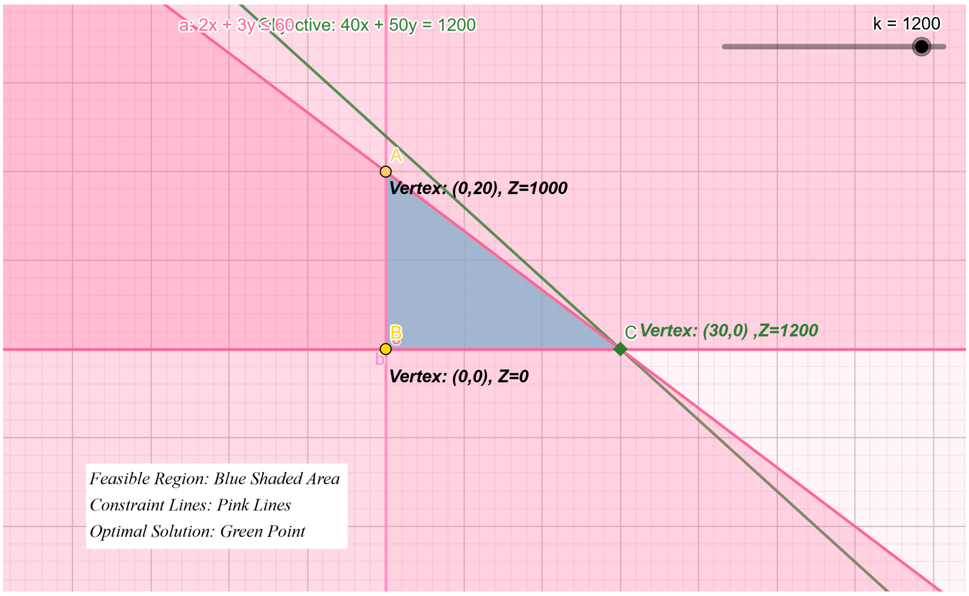

Figure 6: Final Solution

Conclusion

This project demonstrates the application of linear programming using GeoGebra to solve an optimization problem. The process visualizes the feasible region, constraints, and the optimal solution, making the concept more intuitive. The optimal solution is to produce 30 units of product A and 0 units of product B, yielding a maximum profit of $Z = 1200$. GeoGebra's interactive tools provide a clear and efficient way to solve and present linear programming problems, making it an excellent resource for learning and teaching optimization techniques.