Visualizing Linear Programming

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Introduction

Linear programming is a mathematical technique used to determine the best possible outcome (such as maximizing profit or minimizing cost) under a set of given constraints. This powerful method is widely applied in various fields such as manufacturing, transportation, and resource allocation. By using linear programming, businesses and organizations can make optimal decisions based on their available resources.
Linear programming problems are defined by an objective function that needs to be maximized or minimized and constraints that restrict the possible solutions. These constraints are usually represented as inequalities.
This project demonstrates how to solve a linear programming problem using GeoGebra, a free dynamic mathematics software. GeoGebra provides interactive tools for graphing, visualization, and solving mathematical problems. By following the steps outlined in this report, you will learn how to formulate and solve a real-world optimization problem step-by-step. The project focuses on maximizing the profit of a company that produces two products, considering resource constraints.

The Problem

A company produces two products, A and B. Each unit of A requires 2 hours of labor, while each unit of B requires 3 hours of labor. The total labor available is 60 hours. Each unit of A generates $40 in profit, and each unit of B generates $50 in profit. Formulate a linear programming problem to determine how many units of each product the company should produce to maximize its profit.

Breaking down the problem-

- **Objective**: A mathematical representation of the goal to optimize (maximize or minimize).
  The goal is to maximize the total profit, represented by the equation:
  where:
  - x: the number of units of Product A produced.
  - y: the number of units of Product B produced.
- **Constraints**: A set of linear inequalities representing limitations on resources.
  The problem is subject to the following constraints:
  - 1. Labor Constraint: The total labor used cannot exceed 60 hours.
  - 2. Non-negativity Constraints: The company cannot produce a negative number of units.

The solution to this problem will determine how many units of each product the company should produce to maximize profit while satisfying the constraints.
Solving this problem involves identifying a feasible region and finding the optimal solution within this region.
Feasible Region: The graphical intersection of all constraints, representing possible solutions.
Optimization: Identifying the vertex of the feasible region that gives the optimal value of the objective function.

Methodology

We will solve this problem using GeoGebra by following these steps:

1. Formulate the problem by defining the objective function and constraints.
2. Plot the constraints on a graph as inequalities.
3. Identify the feasible region, which satisfies all constraints.
4. Plot the objective function as a line and determine the point of tangency with the feasible region.
5. Compute the profit at each vertex of the feasible region to find the optimal solution.
6. Annotate the graph and clearly highlight the solution.

GeoGebra is used to:

- Graph the constraints.
- Highlight the feasible region.
- Visualize the objective function and find the optimal solution interactively.

Procedure

The steps to solve the problem in GeoGebra are as follows:

1. Define the Problem

- **Objective**: Maximize profit $Z = 40x + 50y$, where x and y represent the units of products A and B, respectively.
- **Constraints**:
  - $2x + 3y \leq 60$ (labor constraint)
  - $x \geq 0$ and $y \geq 0$ (non-negativity constraints)

These equations form the foundation of the solution.

2. Set Up GeoGebra

Open the **Graphing Calculator** in GeoGebra.

1. Launch the GeoGebra Graphing Calculator:Access it online at GeoGebra.org or use the desktop/mobile application.
2. Ensure the axes and grid are visible for better visualization
3. Adjust the graph view to accommodate the range of values for and based on the constraints.

3. Input Constraints

Plot the constraints as inequalities:In the input bar at the bottom, type the constraints one by one

- Enter $2x + 3y \leq 60$ and press Enter. GeoGebra will shade the region below the line.
- Enter $x \geq 0$ and press Enter. GeoGebra will shade the region to the right of the x-axis.
- Enter $y \geq 0$ and press Enter. GeoGebra will shade the region above the y-axis.

The shaded intersection represents the feasible region.

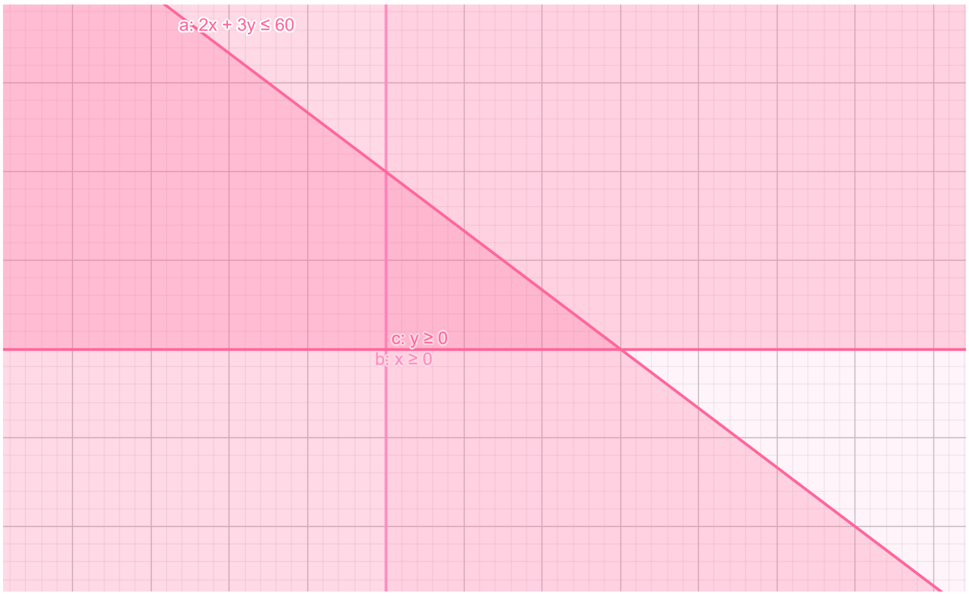

Figure 1: Constraints plotted

4. Identify the Feasible Region

Determine the vertices of the feasible region by finding intersections of the constraint lines. o Use the Intersection Tool in GeoGebra to calculate the intersection points of the lines.

- The lines $2x + 3y \leq 60$, $x \geq 0$, and $y \geq 0$ intersect at three points:
  $(0, 0)$, $(0, 20)$, and $(30, 0)$.
- These vertices represent the corners of the feasible region.
- Label the vertices using the **Point Tool**:
  Rename the points as B, A and C respectively
- Highlight the feasible region:
  Use the **Polygon Tool** in GeoGebra to connect the vertices $(0, 0)$, $(0, 20)$, and $(30, 0)$. The shaded area formed by these points is the feasible region

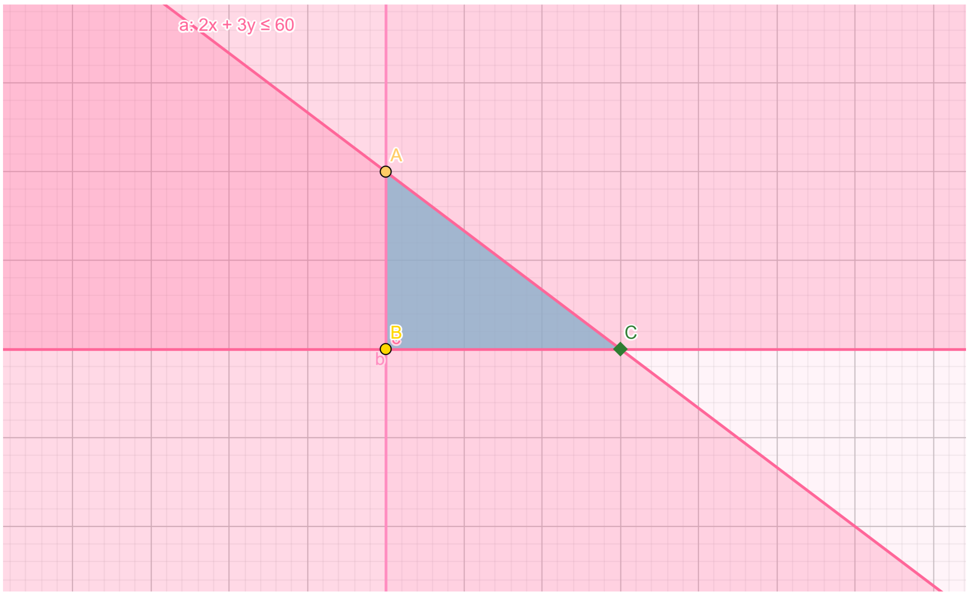

Figure 2: Vertices of the feasible region forming a triangle

5. Enter the Objective Function

Rewrite the profit function $Z = 40x + 50y$ as$k = 40x + 50y$, where k is a constant representing different profit levels.

Use a slider to visualize how $Z = k$ changes as the line moves through the feasible region.

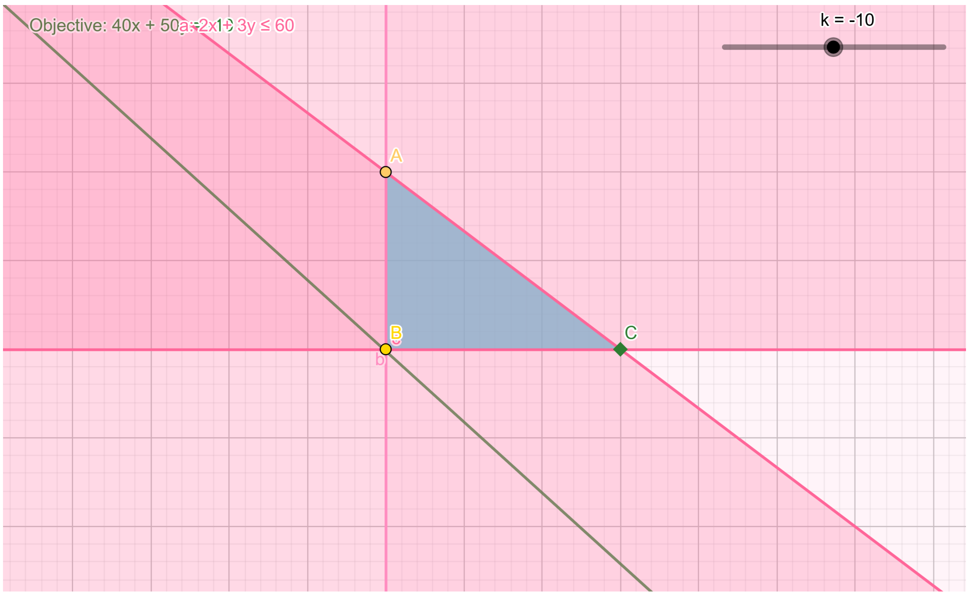

Figure 3: Profit function visualization with slider

6. Evaluate the Objective Function at Vertices

Compute Z (the profit) at each vertex of the feasible region::

- $Z(0, 0) = 0$
- $Z(0, 20) = 1000$
- $Z(30, 0) = 1200$

The maximum profit is $Z = 1200$ which occurs at $(30, 0)$.
This means the company should produce **30 units of Product A** and **0 units of Product B** to maximize profit.

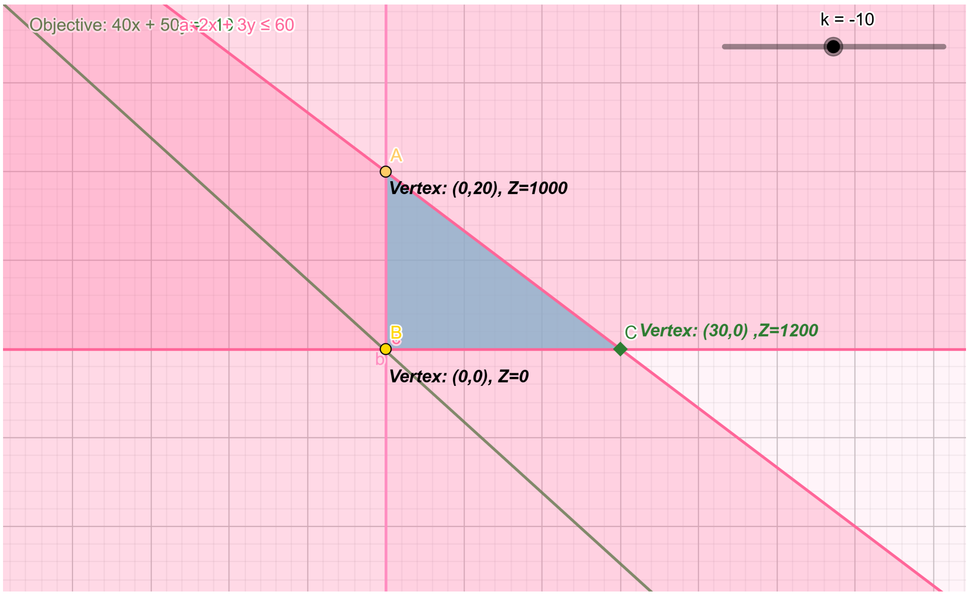

Figure 4: Evaluating Vertices

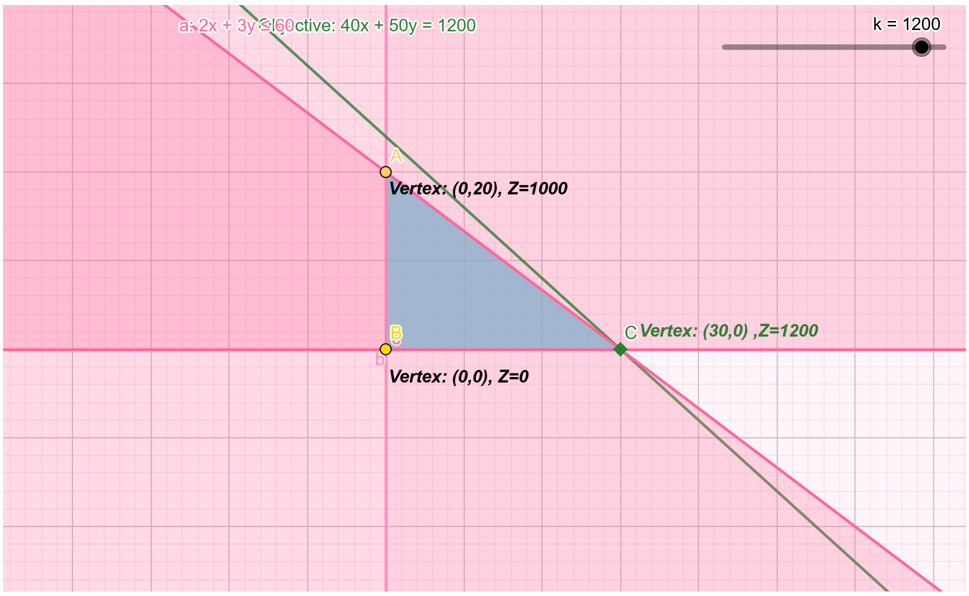

Figure 5: Profit Function Visualization

7. Highlight and Annotate

Shade the feasible region. Label the constraints, vertices, and the optimal solution. Add a legend to explain the graphical elements.

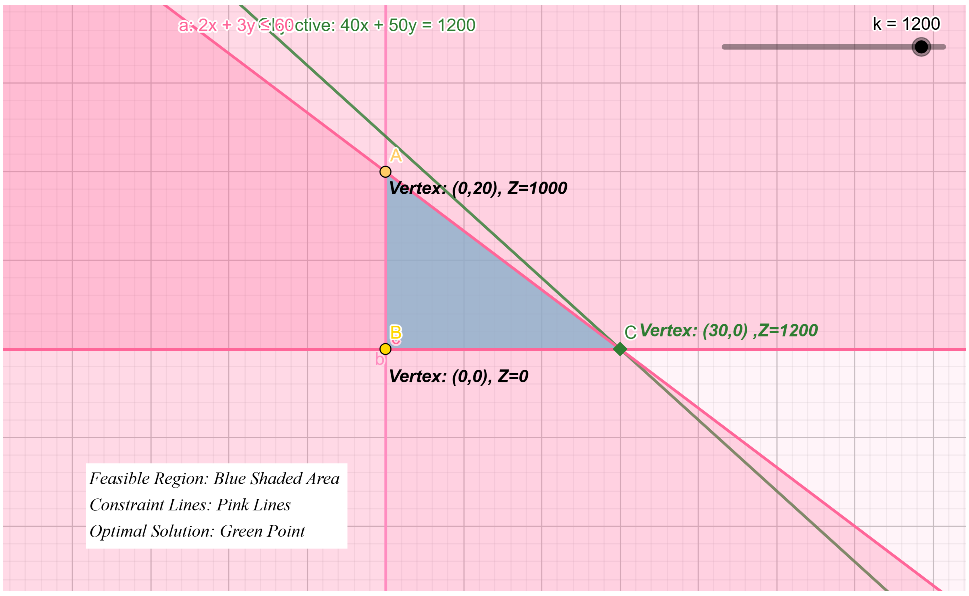

Figure 6: Final Solution

Conclusion

Using GeoGebra, we solved a linear programming problem to determine the optimal production strategy for two products. By plotting constraints, identifying the feasible region, and evaluating the profit function at each vertex, we found that the company should produce 30 units of Product A and 0 units of Product B to achieve a maximum profit of $1200.

GeoGebra's interactive tools made the process intuitive, allowing for clear visualization and precise calculations. This project demonstrates how linear programming can be effectively used to solve real-world optimization problems. Moreover, the methodology used in this project can be adapted to other similar problems in various fields, such as resource allocation, scheduling, and production planning.