Visualizing a Derivative using GeoGebra

Bhavik Dodda

Department of Mathematics,

Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat

1. Aim

Understanding derivatives is a fundamental concept in calculus, but many learners struggle to intuitively grasp its geometric significance. This project aims to leverage GeoGebra to create an interactive tool that helps to visualize and understand the derivative f'(x) as the slope of the function f(x) at a given point x=l.

2. Introduction

The GeoGebra interactive tool will feature a zoom widget integrated with a graphing interface. Users can select any point x=l on the curve and gradually zoom into the point (l,f(l)), observing how the curve locally resembles a straight line. This visualization reinforces the idea that as the zoom level increases, the slope of this straight line converges to f'(l).

This project offers an intuitive and interactive way to explore the concept of derivatives, and for learners to self discover the limit definition of a derivative!

3. Usage

An end users of the interactive graph (derivative.ggb) would have control over the following parameters:

- Set the function that they want to visualize the derivative of, in f(x)
- Set the x coordinate x=l at which point they want to get the derivative: f’(l)
- Variable **Size** sets the magnifying glass’s radius
    - The smaller the value of **Size,** the larger would be the zoom ratio.

There are 3 approximations of the derivative displayed with respect to the zoom magnification factor and also the actual derivative at point x=l.

Approx1: 0.3927154923936
Approx2: 0.3942505106196
Approx3: 0.3911804741675
Actual: 0.3927134923936

4. Methodology

(Additionally, refer to the attached video **derivative.mp4**)

i. Defining function f(x) and the point on the function f(x) at x=l is **point C=(l,f(l))**. "l" varies between Images 1 and 2.

ii. **Defining 2 circles**:
   a. Magnifying glass **c** over **point C** has a radius defined by **Size (Slider)**
   b. Defining the magnified view window **eq1** where **Point P=(A,B)** is its center

iii. **Enlarging the curve:** The curve f(x) that is inside this magnifying glass **c** has to be re-displayed within the magnified view window. This is done using transformations:
   a. Let the re-displayed function within the magnified view be h(x)
   b. A point (x,y) within **c** would map to the linear transformation (X,Y)=(5/Size * (x-l) + A, 5/Size * (y-f(l)) + B) centered at **point P**
   c. Y corresponds to h(X). Therefore h(X)=Zoom(f(X))
      i. Where **Zoom(inp) = 5/Size*(inp-f(l))+B**
   d. Here, the function h(X) is in terms of X, but we would need it in terms of x to plot the function. Therefore, h(x)=Zoom(f(Zoominv(x)))
      i. Where x is solved in terms of X, **Zoominv(inp) = Size/5*(inp-A) + l**
   e. Finally, limiting the function h(x) graph within the **eq1** using the if condition. **If((x-A)^2+(h(x)-B)^2<5^2,h(x))**, this ensures that the function gets displayed only within the circle.

iv. **Linking the 2 circles**: For the users to understand that the magnified view window and magnifying glass are linked together, we need to draw tangents between the circles **c** and **eq1.**
   a. Tangents can be drawn by utilizing the Construct menu from the Tools tab
   b. Two of the 4 tangents Geogebra automatically produces are relevant for our purpose: **i, j.**
   c. Find intersections of **i, j** with **c** and **eq1**. Tangents i, j intersect with circles c, eq1 at 4 points. Let E,G be the end points of tangent **i**, and D,H be the end points of tangent **j.**
   d. Limit the line segments by utilizing the if condition and the x(…) function
      i. If(x(E) < x < x(G), i(x))
      ii. If(x(D) < x < x(H), j(x))

v. **Tangent line**: Adding an additional tangent line **q(x)** in the magnified view window for reference to curve h(x). We use the point-slope form of the line to find the equation of q(x)
   a. q(x)=f'(l)(x-A)+B is the line with slope=f'(l), passing through P=(A,B)

vi. **Limit approximations**: There are 3 kinds of limit approximations of a derivative, which converge to f'(l) as Size→0 (See images 3,4,5,6). These are computed and displayed within **text1**
   a. Slope between (l+size,f(l+size)) and (l-size,f(l-size)) defined within LimSlope1
      i. L_{imSlope1}(x)=((f(l+x)-f(l-x))/(2 x))
   b. Slope between (l,f(l)) and (l+size,f(l+size)) defined within LimSlope2
      i. L_{imSlope2}(x)=((f(l+x)-f(l))/(x))
   c. Slope between (l-size,f(l-size)) and (l,f(l)) defined within LimSlope3
      i. L_{imSlope3}(x)=((f(l)-f(l-x))/(x))

vii. **text1** also contains the actual derivative at point l: f'(l), for comparison.

5. Conclusion

The project leverages Geogebra's interactive capabilities to provide an intuitive understanding of derivatives by visually linking the concept of slope to the tangent. The graph allows the user to vary the size of the magnifying glass and see the three limit approximations of the derivative approach the actual derivative. This makes the abstract limit definition of the derivative intuitive, making the user self-discover the standard definition and other parallel ways to redefine the derivative.

Images

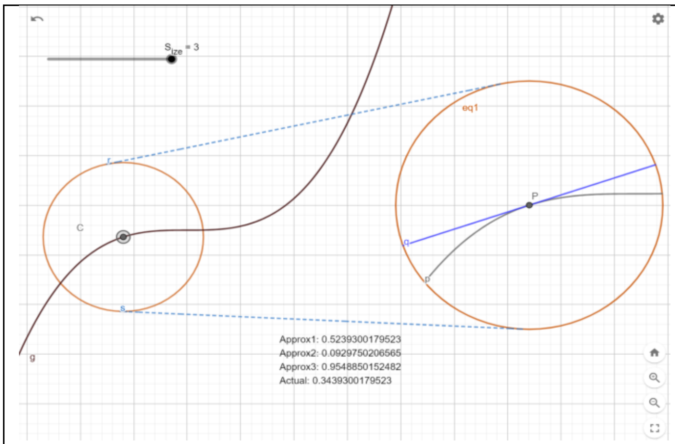

Image 1

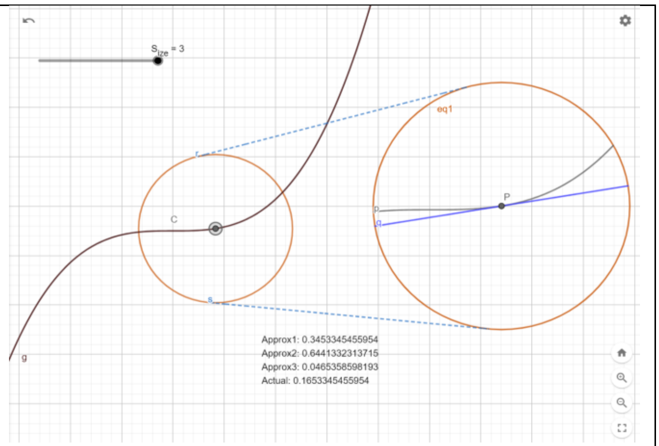

Image 2

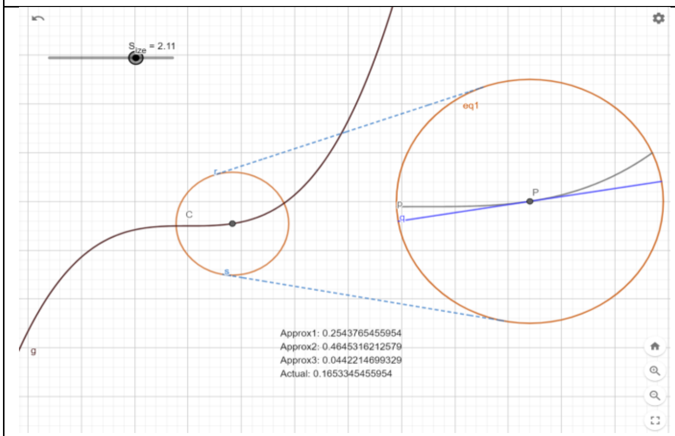

Image 3

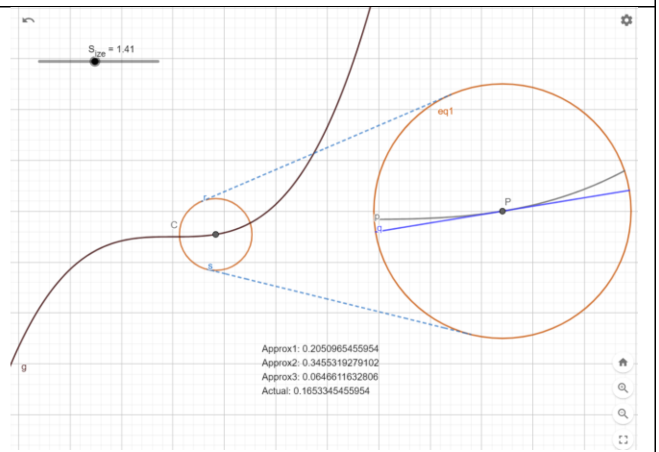

Image 4

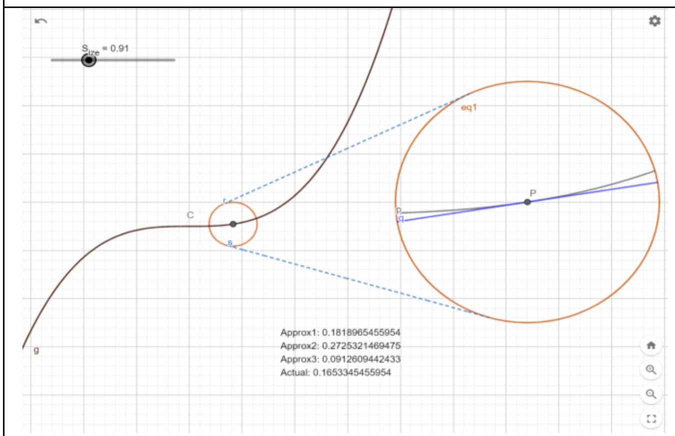

Image 5

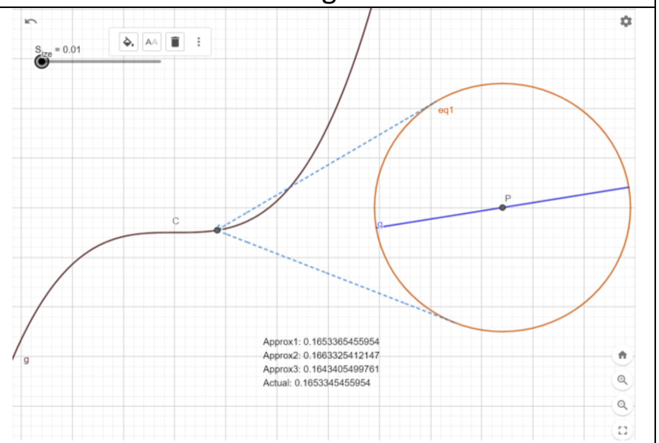

Image 6