Formation, Rotation and Revolution of Conics in 3D

Tanushree. L
Department of Mathematics,
Avinashilingam Institute for Home Science & Higher Education for Women, Coimbatore – 641043

1. Introduction

Conics are familiar to us because of its wide use in all fields. Study of conics opened innovations in construction of buildings, space science, medicine etc. Conic sections have been studied by the ancient Greeks as an important mathematical concept. In early 320 BCE, Greek mathematicians such as Menaechmus, Apollonius, and Archimedes were fascinated by these curves. Apollonius wrote an entire eight-volume treatise on conic sections in which he was able to derive a specific method for identifying a conic section using geometry. The conic sections are generated by the intersections of a plane with one or two nappes of a cone.

2. Methodology

2.1 Formation of conics

Conic sections are formed by intersecting a plane with a cone. A cone has two identically shaped parts called nappes. One nappe is either of the two identical portions of a cone that meet at the vertex. Each portion is equivalent to a right circular. A right circular cone can be generated if the plane is parallel to the x-axis, then that conic section is a circle. If the plane intersects one nappe at an angle to the axis (other than **90°**), then the conic section is an ellipse. If the plane is intersecting near the bases, the conic section is a parabola. If the plane is parallel to the *y*-axis, then the conic section is a hyperbola. In GeoGebra using 3D graphics we can demonstrate the formation of conic sections.

2.2 Rotation of conics

The conic sections such as circle, ellipse, parabola and hyperbola can be rotated with respect to x axis or y axis. In GeoGebra, we can visually see the rotation of these conic sections with respect to x axis.

2.3 Revolution of conics

The conic sections such as circle, ellipse, parabola and hyperbola can be shown as revolution of surfaces with respect to x axis or y axis. In GeoGebra, we can visually see the revolution of these conic surfaces with respect to x axis.

3. Algebra

3.1. Formation of Conics

Using **GeoGebra Classic 6,** let us create conic sections in the **3D Graphics.**

Step - 1 : Open a new GeoGebra window.
Create sliders a, b, c, p, q and r using the Slider tool.

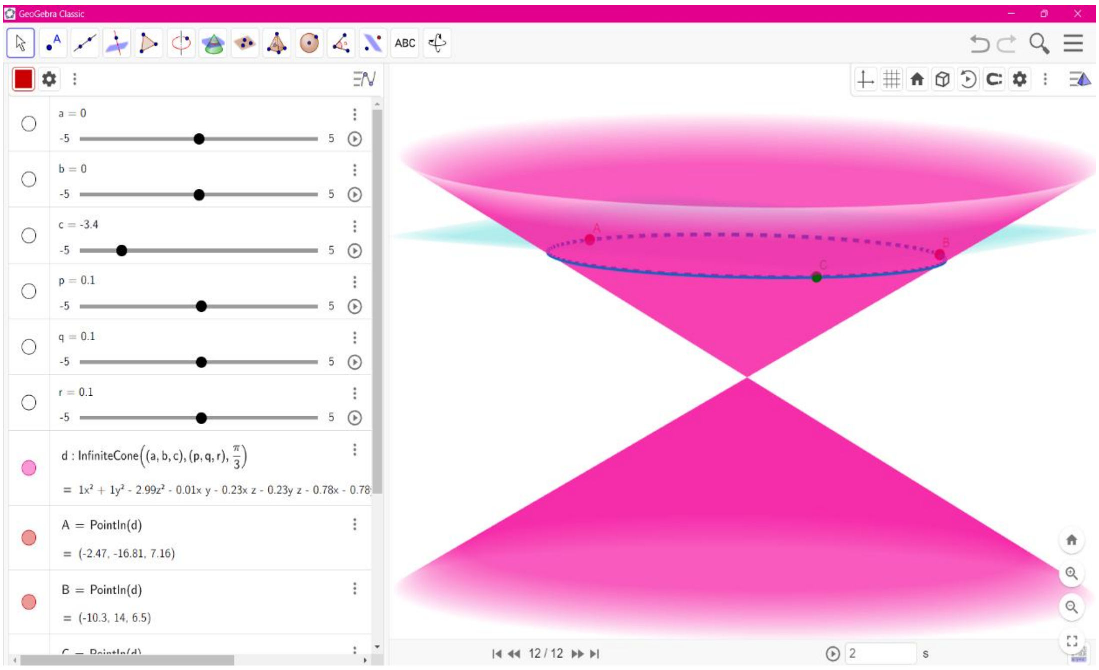
Step - 2 : Type in the input bar as “**d: InfiniteCone((a,b,c),(p,q,r),((π)/(3)))**” to create an infinite cone.

Step - 3 : To create an intersection plane in the infinite cone, use the “Plane through 3 points” tool and Click any three points say A, B and C in the infinite cone.

Step - 4 : We can see different conic sections by dragging the points A, B and C of the intersection plane.

Step - 6 : To view the conic sections clearly in the 3D view let us type in the input bar as “**f: IntersectPath(p_{1},d)**” where $\mathbf{p_1}$ is the intersection plane and **d** is the infinite cone.

Step - 7 : By dragging A, B and C points we can see the four types of conic sections- circle, ellipse, parabola and hyperbola in the 3D view.



3.2. **Rotation of conics**

Step - 1 : Open a new GeoGebra window.
Create a circle with radius 5 units using circle: Center & Radius tool.

Step - 2 : Click a point A in the Graphics view and type the radius as 5 units, a circle with center A and radius 5 units is created.

Step - 3 : Create an ellipse using the ellipse tool and by clicking three points in the Graphics view according to convenience.

Step - 4 : Create a parabola using the parabola tool by clicking the x-axis and a point in the Graphics view.

Step - 5 : Create a hyperbola using hyperbola tool. Click and select the three points where hyperbola should be placed.

Step - 6 : Let us create an angle slider for angle as α so that the conics will rotate with respect to α.

Step - 7 : To rotate the conics created, use the Rotate command in the Graphics view.

Step - 8 : Type in the input bar as “c': Rotate(c,α)” where c is the circle created and c’ is the circle rotated by angle α.

Step - 9 : Type in the input bar as “d': Rotate(d,α)” where d is the ellipse created and d’ is the ellipse rotated by angle α.

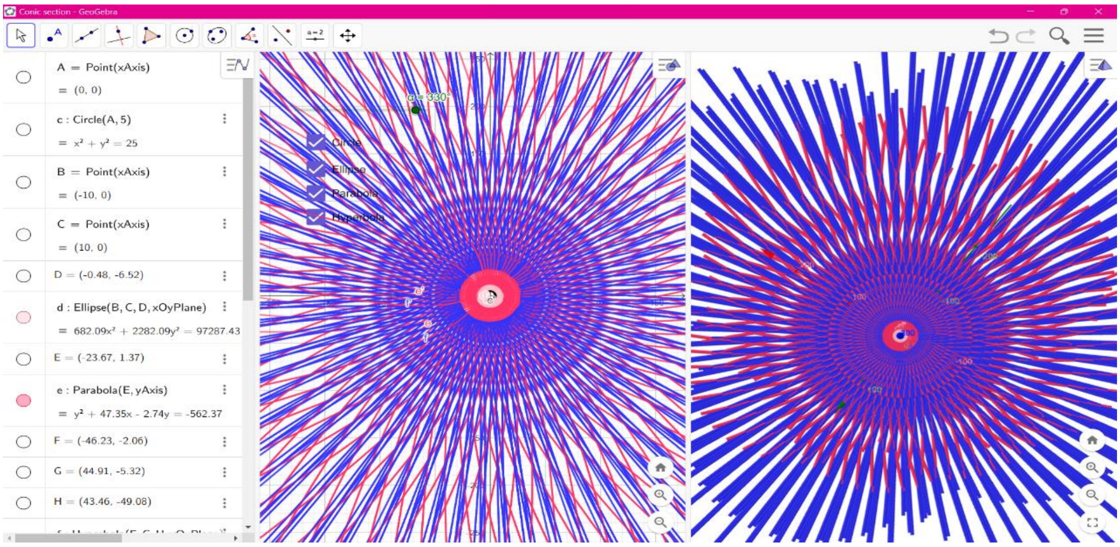
Step - 10 : Type in the input bar as “e': Rotate(e,α)” where e is the parabola created and e’ is the parabola rotated by angle α.

Step - 11 : Type in the input bar as “f': Rotate(f,α)” where f is the hyperbola created and f’ is the hyperbola rotated by angle α.

Step - 12 : To trace the rotating conics, select the trace on option from the settings of each conic rotated by angle α given in the input bar.

Step - 13 : Create check boxes to show or hide each conic using check box tool in the Graphics view.

Step - 14 : By selecting animation option of the slider of angle α and selecting the conic from the check box we can view the rotation of conics in both the Graphics and 3D view.



3.3. **Revolution of conic surfaces**

Step - 1 : Open a new GeoGebra window.
Create a circle with radius 5 units using circle: Center & Radius tool.

Step - 2 : Click a point A in the Graphics view and type the radius as 5 units, a circle with center A and radius 5 units is created.

Step - 3 : Create an ellipse using the ellipse tool and by clicking three points in the Graphics view according to convenience.

Step - 4 : Create a parabola using the parabola tool by clicking the x-axis and a point in the Graphics view.

Step - 5 : Create a hyperbola using hyperbola tool. Click and select the three points where hyperbola should be placed.

Step - 6 : Let us create a slider for angle as α so that the conics will rotate with respect to α.

Step - 7 : For the revolution of the conics about an axis use Surface of Revolution tool in the 3D view and then click the curve of the conic in the 3D Grahics view to bring about the revolution.

Step - 8 : Input will be automatically generated after clicking the tool and the object. We can also type the inputs in the input bar and press Enter as follows.

Step - 9 : Type in the input bar as “c'=Surface(c,α,xAxis)” where c is the circle created and c’ is the revolution of circle with angle α.

Step - 10 : Type in the input bar as “d'=Surface(d,α,xAxis)” where d is the ellipse created and d’ is the revolution of ellipse with angle α.

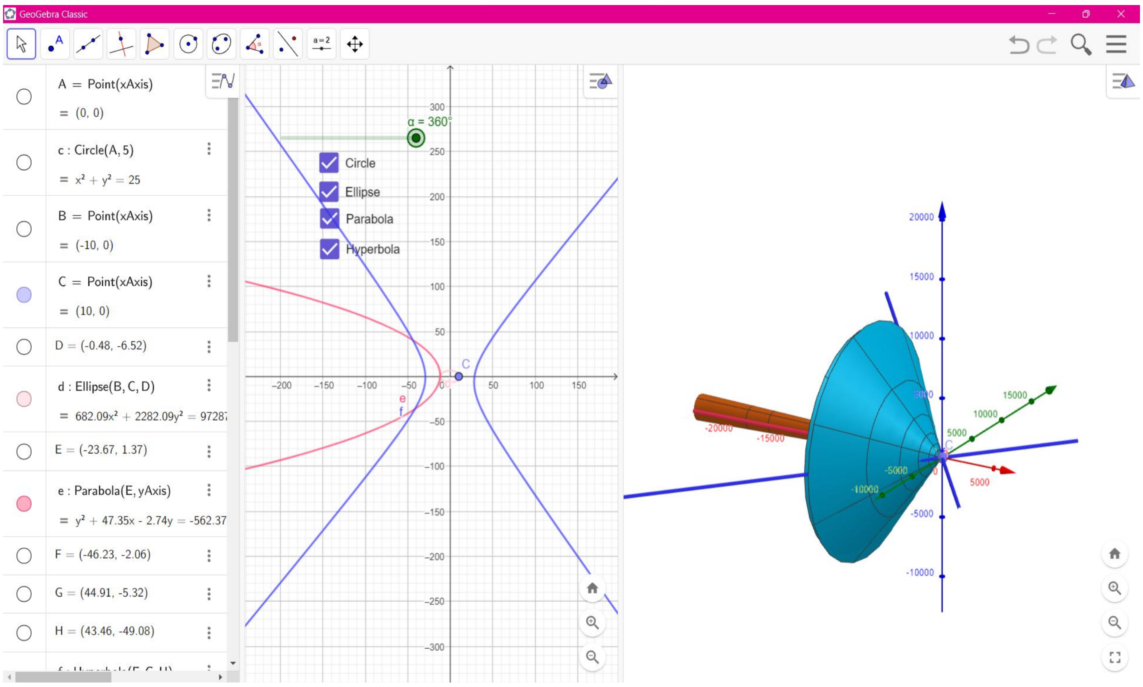
Step - 11 : Type in the input bar as “e'=Surface(e,α,xAxis)” where e is the parabola created and e’ is the revolution of parabola with angle α.

Step - 12 : Type in the input bar as “f'=Surface(f,α,xAxis)” where f is the hyperbola created and f’ is the revolution of hyperbola with angle α.

Step - 13 : To trace the revolution of the conics, select the trace on option from the settings of each input of revolution of the conics given in the input bar.

Step - 14 : Create check boxes to show or hide each conic using check box tool in the Graphics view.

Step - 15 : By selecting animation option of the slider of angle α and selecting the conic from the check box we can view the revolution of the conics in both the Graphics and 3D view.



4. Conclusion

We show that the formation, rotation and revolution of the conic sections could be visually viewed by GeoGebra software. In GeoGebra, we perform this using various tools in the software which enables the users to experience conics in 3D Graphics.