L’Hôpital’s Rule for Evaluating Limits

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1. Introduction:

Mathematics presents intriguing challenges when it comes to evaluating limits. Indeterminate forms in functions have puzzled mathematicians for ages. Fortunately, L’Hôpital’s Rule, named after the French mathematician Guillaume Francois Antoine, Marquis de L’Hôpital, offers a powerful solution. By using derivatives and calculus, this rule provides a systematic approach to handle limits involving function ratios. In GeoGebra Classic 5, we have solved limit problems by defining functions and their derivatives. This is done by calculating the values at a specific point on the graph.

2. Methodology:

The methodology employed in this project focuses on the systematic application of L’Hopital’s Rule to evaluate limits exhibiting indeterminate forms.

The steps involved in this methodology are as follows:

1. **Identify the indeterminate form:** Let us begin by identifying the limit expression that exhibits an indeterminate form, such as “0/0” or “∞/∞”. These forms indicate the potential applicability of L’Hôpital’s Rule.

2. **Simplify the expression:** To apply L’Hôpital’s Rule, rewrite the limit expression as a ratio of two functions, f(x) and g(x), where both functions approach zero or infinity as x approaches a specific value.

3. **Differentiate numerator and denominator**: Calculate the derivatives of both f(x) and g(x) with respect to x. This step requires knowledge of differentiation rules and techniques.

4. **Evaluate the limit of the ratio**: Compute the limit of the derivative of the numerator divided by the derivative of the denominator. This new limit expression may still be in an indeterminate form, which allows for further iterations of L’Hôpital’s Rule.

5. **Repeat the process if necessary**: If the limit expression obtained in the previous step is still indeterminate, continue differentiating the numerator and denominator and recalculating the limit until an explicit value or an alternative form is obtained.

6. **Check for conditions of applicability**: Ensure that the conditions for applying L’Hôpital’s Rule are satisfied. These conditions typically involve the functions being differentiable in a neighborhood of the value under consideration.

7. **Determine the final limit**: Once a non-indeterminate form is obtained, evaluate the limit using standard techniques, such as direct substitution or algebraic simplification.
8. **Validate the result**: After obtaining the limit value, verify its accuracy by checking the behavior of the original function through additional methods, such as graphing or numerical approximation.

3. Steps:

Step – 1: Open GeoGebra interface and create a new file.

Step – 2: Using the input bar, define the functions f(x) and g(x) as f(x) = x^2-x and g(x)=x-1, respectively. Find the derivatives of f(x) and g(x), denoted as f`(x) and g`(x), which are f`(x)=2x-1 and g`(x)=1, respectively.

Step – 3: Create a slider named “c” with the value of 1. This slider represents the value that x approaches as it tends to c.

Step – 4: Calculate the values of a, b, d and e by evaluating the functions and derivatives at the given value of c:

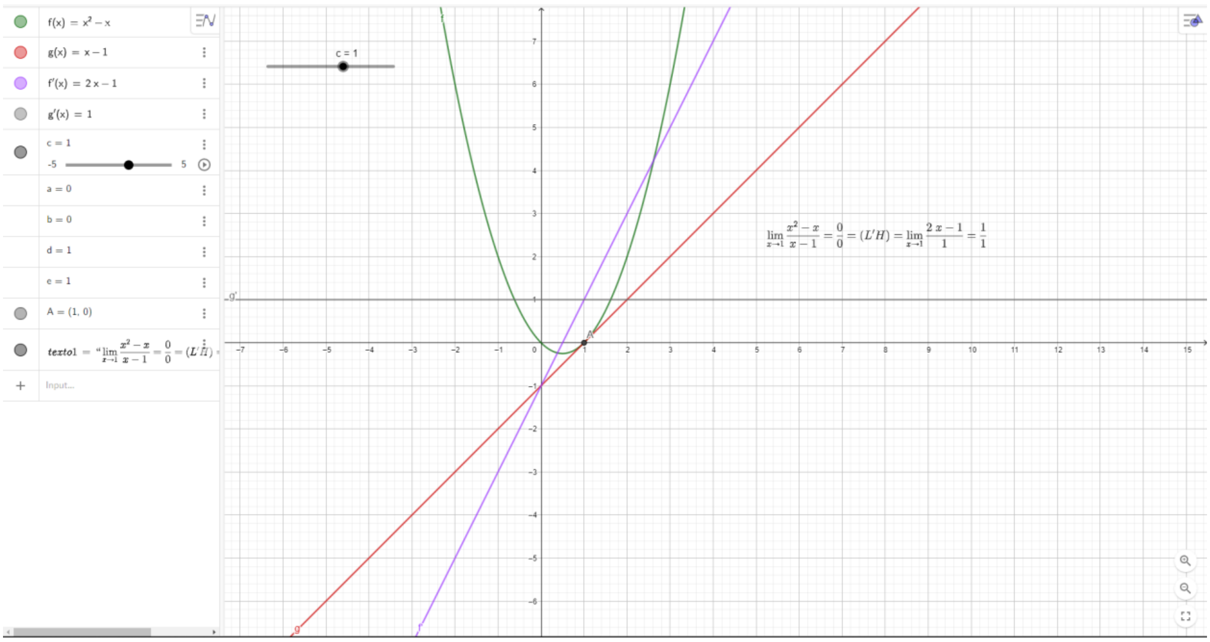
Set a as the result of evaluating f(c).

Set b as the result of evaluating g(c).

Set d as the result of evaluating f`(c).

Set e as the result of evaluating g`(c).

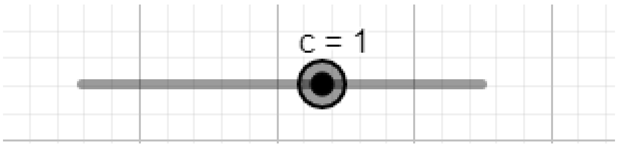
Step – 5: Create a point A at coordinates (c,0) to represent the point on the graph.

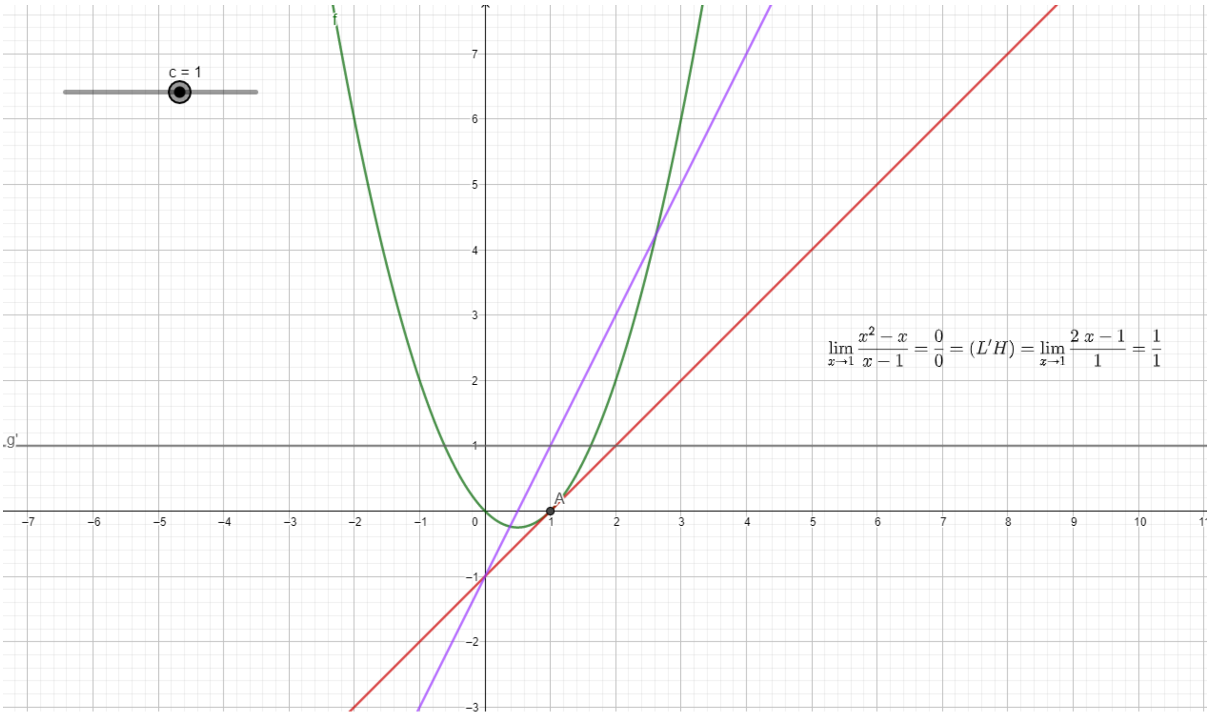


Step – 6: Add a text box to display the L’Hôpital’s rule and the limit expression. Write the limit expression "\lim {x\to c}\frac{a}{b}=(L’H) = \lim_{x\to c}\frac{f`}{g`} = \frac{d}{e}” to represent the equation.

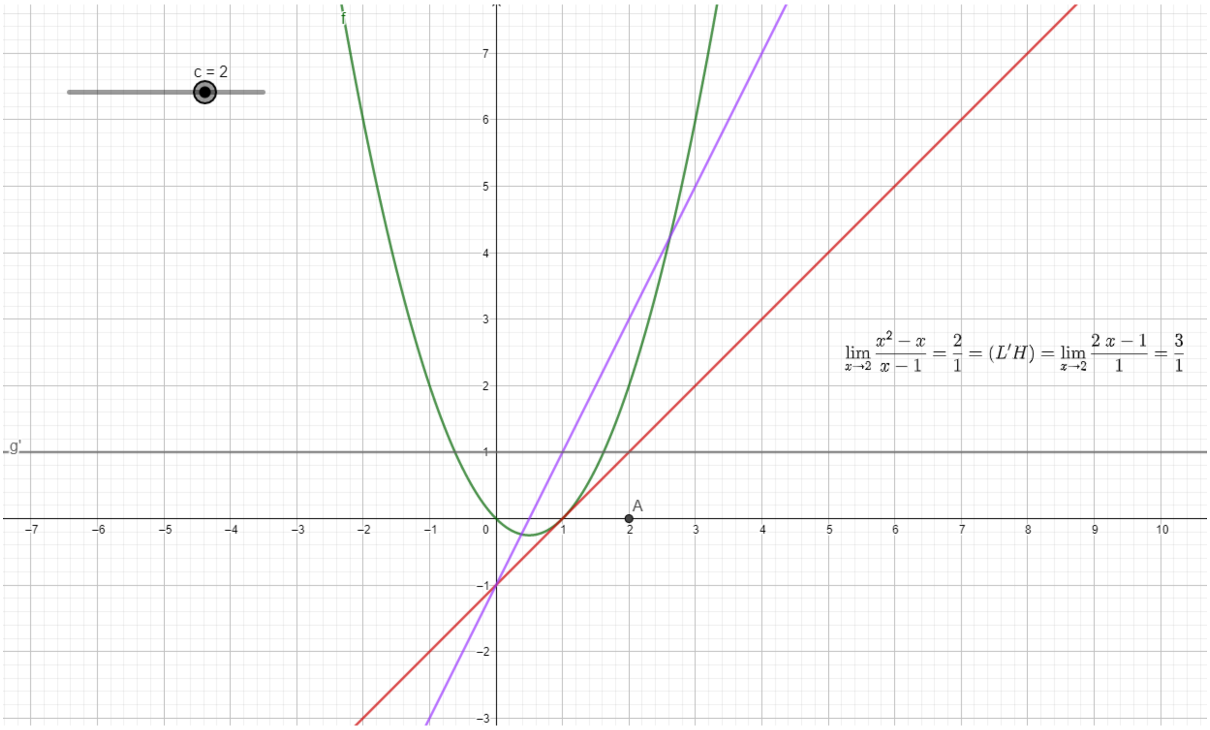
$$\lim_{x \to 1} \frac{x^2 - x}{x - 1} = \frac{0}{0} = (L'H) = \lim_{x \to 1} \frac{2\,x - 1}{1} = \frac{1}{1}$$

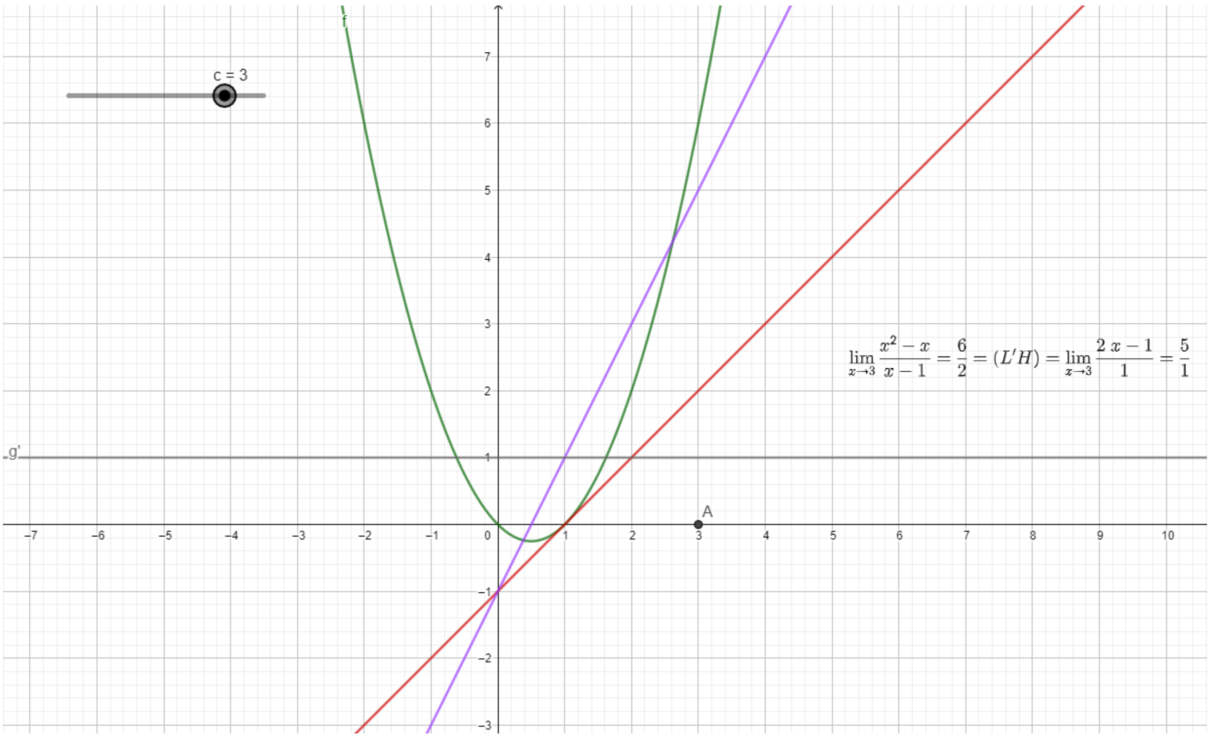
Step – 7: Interact with the graph by moving the slider “c” and observe how the graphs and the text box change accordingly.





Now move the slider and observe the limits at respective values,





4. Conclusion:

Therefore, the GeoGebra project effectively demonstrated L'Hôpital's Rule for evaluating limits. By defining functions, calculating derivatives, and visualizing the limit expression through graphs, we gained a clear understanding of how this rule works in calculus.