

Calculate the Riemann Sums using GeoGebra

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1. Introduction

Riemann sums plays an inevitable role in Real Analysis for finding Riemann integral. Riemann sums are used to determine the finite integral by approximate values. Riemann sums were discovered by nineteenth century German mathematician Bernhard Riemann. Left Riemann sum, right Riemann sum and middle Riemann sum are the three most common Riemann sums. To find these Riemann sums we have used methods such as left-hand rule, right hand rule and midpoint rule.

In GeoGebra I have plotted graphs to show different Riemann sums and calculated their values which are approximately equal to the definite integral values.

2. Methodology

2.1. Left hand rule

On each subinterval, evaluate the function at the left endpoint and make the rectangle to that height. This method is called as left-hand rule and it is used to calculate left Riemann sum. In GeoGebra software we have separate command to calculate left Riemann sum using this method. The command used is “leftSum” In the input bar we type as “leftSum(f,a,b,n)” where f is the function, a is the lower limit, b is the upper limit and n is the number of rectangles.

2.2. Right hand rule

On each subinterval, evaluate the function at the right endpoint and make the rectangle to that height. This method is called as right-hand rule and it is used to calculate right Riemann sum. In GeoGebra software we do not have separate command for right Riemann sum so we use Δx , where Δx is the width of each rectangle.

$$\Delta x = \frac{b - a}{n}.$$

We use leftSum command with Δx to get right Riemann rule. In the input bar type “LeftSum(f(x + Δx), a, b, n)” where f is the function, a is the lower limit, b is the upper limit and n is the number of rectangles.

2.3. Midpoint point rule

On each subinterval, evaluate the function at the midpoint and make the rectangle to that height. This method is called as midpoint rule and it is used to calculate middle Riemann sum. In GeoGebra software we do not have separate command for middle Riemann sum so we use Δx , where Δx is the width of each rectangle. We use leftSum command with Δx to get

middle Riemann rule. In the input bar type “LeftSum($f(x + \Delta x / 2)$, a, b, n)” where f is the function, a is the lower limit, b is the upper limit and n is the number of rectangles.

3. Procedure to plot the Riemann sums in GeoGebra.

Step - 1 : Type the function $f(x) = x^2$ in the input text box tool.

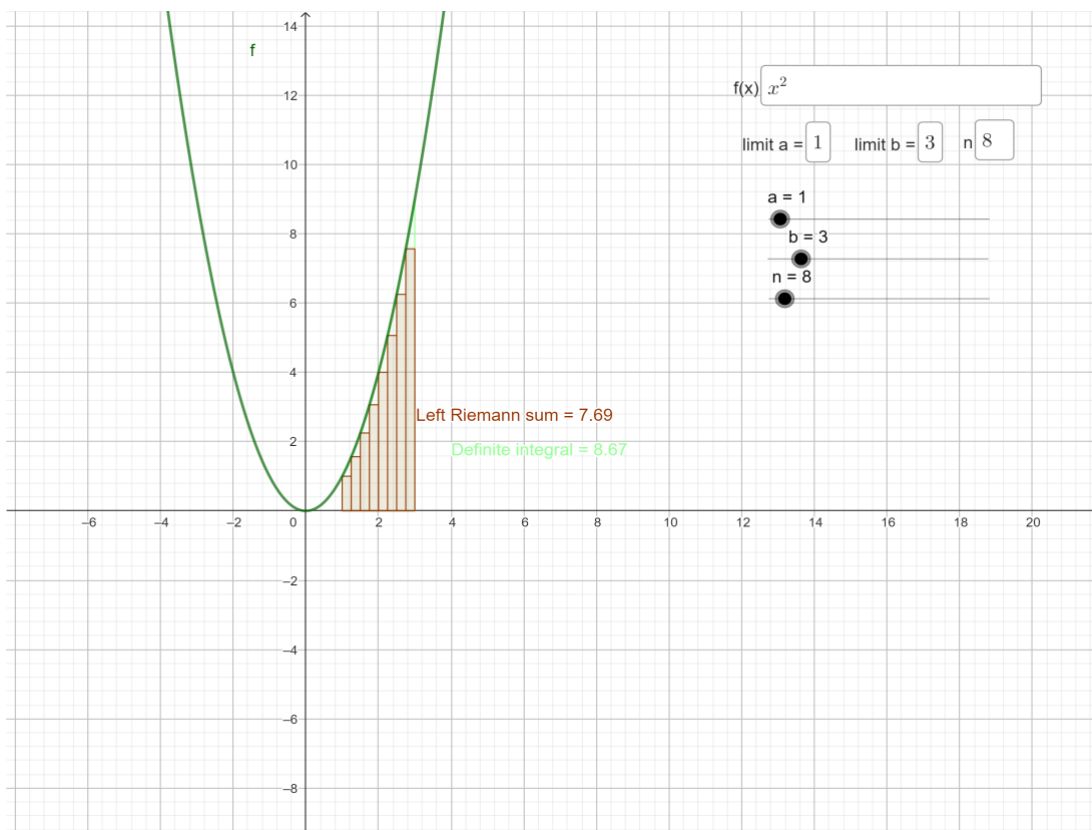
Step - 2 : Create three sliders a, b and n using the slider tool. We can choose lower limit, upper limit and number of rectangles as per our convenience.

Step - 3 : Here slider a is used for lower limit, b for upper limit and n for number of rectangles.

Step - 4 : Using input box tool let us create input boxes by clicking inside the Graphics view. In linked objects let us select the input as f(x), a, b, n.

Step - 5 : Type integral command in input bar as “ integral(f ,a , b)” and press Enter to show the definite integral.

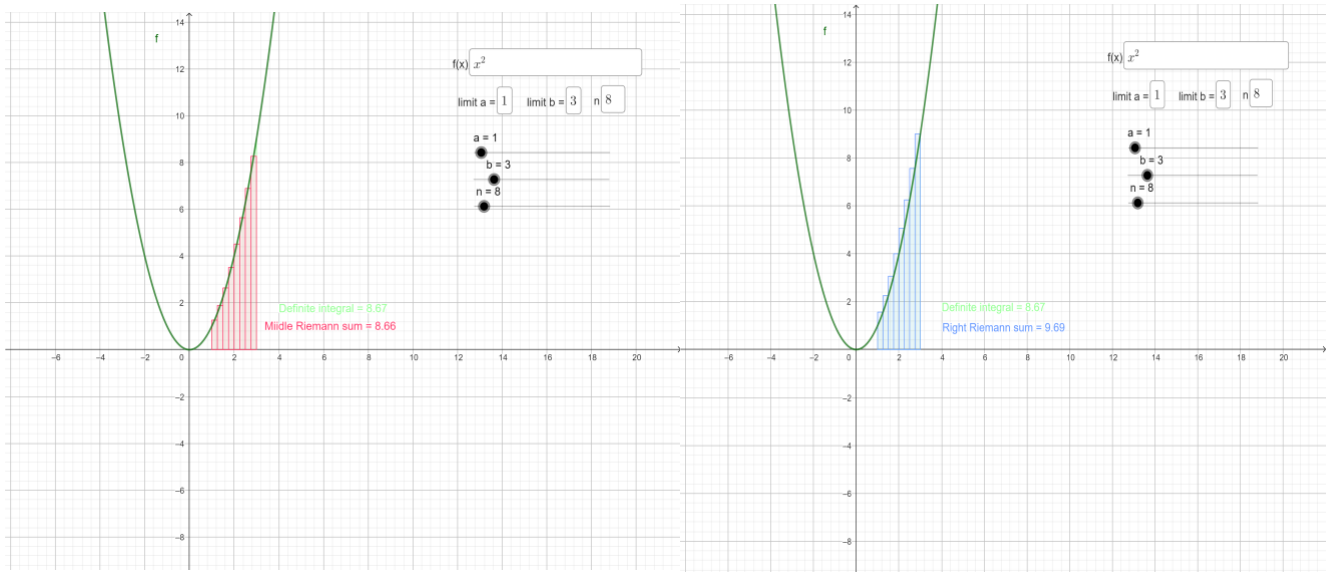
Step - 6 : To find the left Riemann sum use leftsum command. Type the input as ”leftsum(f,a,b,n)” .



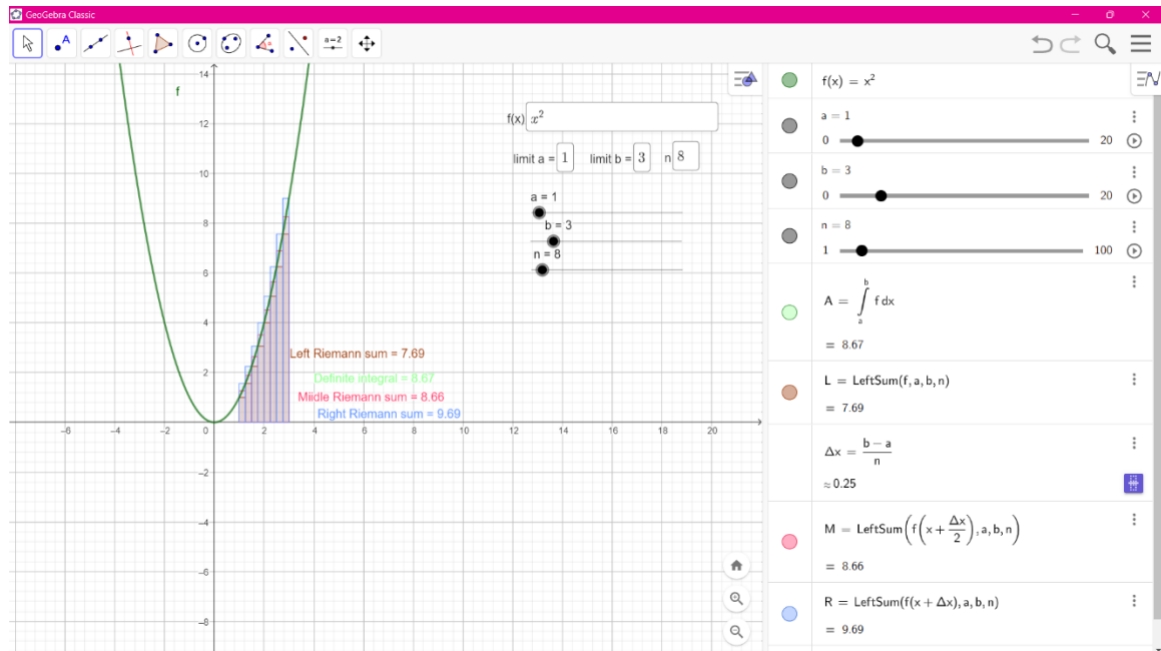
Step - 7 : In GeoGebra we do not have commands to find right Riemann sum and middle Riemann sum so we use Δx . Where Δx is the width of each rectangle.

Step - 8: To find Δx by type “ $\Delta x = (a-b)/n$ ” in the input bar.

Step - 9 : To find middle Riemann sum and right Riemann sum, leftsum command is used.



Step - 10 : To find middle Riemann sum type input as “LeftSum($f(x + \Delta x / 2)$, a , b , n)” and to find right Riemann sum as “LeftSum($f(x + \Delta x)$, a , b , n)” .



Step - 11 : Drag sliders to see differences in the graph.

Step - 12 : You can colour the variables to differentiate them in the graph.

4. Conclusion

In this project I have used GeoGebra to show the resultant graph of the theoretical values of Riemann sums as computational visual outputs. The steps used to plot the Riemann sums will make us to learn and understand calculations of Riemann sums.